Money Talks and Matters:

Three Essays on the Theory of Monetary Policy

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Abstract

How should central banks conduct and communicate their policies to serve the goal of stabilizing the macroeconomy? This thesis – consisting of three self-contained essays on dynamic macroeconomics – is mainly intended as a progress report on exploring the normative aspect of monetary policy.

The main result of the first essay is, that in the presence of idiosyncratic risk, the public revelation of information about uncertain aggregate outcomes can be detrimental. By announcing informative signals on non-insurable aggregate risk, the policy maker distorts agents' insurance incentives and increases the riskiness of the optimal allocation that is feasible in self-enforceable arrangements. We consider a monetary authority that may reveal changes in the inflation target, and document that the negative effect of distorted insurance incentives can very well dominate conventional effects in favor for the release of better information.

In the second essay, we study optimal monetary policy with the nominal interest rate as the single policy instrument. Firms set prices in a staggered way without indexation and real money balances contribute separately to households' utility. The optimal deterministic steady state under commitment is the Friedman rule for a broad range of parameters. Optimal monetary policy in the short run is then characterized by stabilization of the nominal interest rate instead of inflation stabilization as the predominant principle.

In the third essay, I examine whether the existence and the timing of real balance effects contribute to the determination of the absolute price level. As the main novel result, I show that there exists a unique price level sequence that is consistent with an equilibrium under interest rate policy, if beginning-of-period money yields transaction services. Predetermined real money balances can then serve as a state variable, implying that interest rate setting should be passive – a violation of the Taylor-principle.

Keywords:
Social value of information, Transparency, Optimal monetary policy, Real and nominal determinacy

Zusammenfassung

Wie sollten Zentralbanken Geldpolitik gestalten und der Öffentlichkeit kommunizieren, um die Ökonomie bestmöglich zu stabilisieren? Diese Dissertation, bestehend aus drei selbständigen Essays in dynamischer Makroökonomik, widmet sich in erster Linie dem normativen Aspekt von Geldpolitik.

Das Hauptresultat im ersten Essay ist, dass bei idiosynkratischen Risiko die öffentliche Bekanntgabe von Informationen zu aggregierten Risiko einen negativen Effekt auf die soziale Wohlfahrt haben kann: durch die Veröffentlichung von Informationen zu nicht-versicherbaren aggregierten Risiko werden die Versicherungsanreize der Individuen verzerrt und damit das individuelle Konsumrisiko erhöht. Als eine Anwendung, analysieren wir die Situation einer Zentralbank, die die Möglichkeit hat, Veränderungen in ihren Inflationszielen anzukündigen und dokumentieren, das der negative Effekt der verzerrten Versicherungsanreize konventionelle positive Aspekte der Ankündigung überwiegt.

In zweiten Essay untersuchen wir optimale Geldpolitik in Falle von nominalen Rigiditäten und einer Transaktionsfriktion. In einem Standardmodell, Money-in-the-Utility function, zeigen wir, dass das langfristige Optimum durch die Friedmansche Regel gegeben ist. Daraus folgt für die kurze Frist, dass das Primat von Geldpolitik auf die Stabilisierung der Zinsen und nicht auf Inflationsstabilisierung ausgelegt sein sollte.

Im dritten Essay untersuche ich, ob die Existenz und die Terminierung von Realkasseneffekten eine wichtige Rolle für die Determiniertheit des allgemeinen Preisniveaus spielen. Als wichtigstes neues Resultat zeige ich, dass auch bei Zinspolitik ein eindeutiges Preisniveau bestimmt werden kann, wenn die Geldmenge zu Beginn der Periode in Transaktionen verwendet wird. Unter diesen Umständen, hat prädeterminiertes reales Geld die Funktion einer Zustandsvariable und die Zinspolitik sollte passiv sein, um eindeutige, stabile und nicht-oszillierende Gleichgewichtssequenzen zu erreichen.

Schlagwörter:
Sozialer Nutzen von Information, Transparenz, Optimale Geldpolitik, Nominale und reale Determiniertheit

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1 Introduction

1.1 Scope of the study

Interest rate decisions made by the Federal Reserve System (Fed), the European Central Bank (ECB), or other prominent central banks are given much attention even by the general public. Thus, there is a strong interest to understand how monetary policy decisions influence the developments of inflation, employment and GDP, i.e. the monetary transmission mechanism. Consequently, a question of equal importance – given a particular transmission mechanism of monetary policy – is: how should central banks conduct and communicate their policies to serve the goal of stabilizing the macroeconomy? This thesis is mainly intended as a progress report on exploring the normative aspect of monetary policy.

This study focuses on three issues in dynamic macroeconomics and monetary economics. The first one is concerned with optimal disclosure policies of policy makers in particular central banks. In particular, Vadym Lepetyuk and me investigate in chapter 2 whether these institutions should reveal their future policies and economic projections to the public or whether they should be closemouthed. While central banks in the past where well known for their secrecy and mystique, things have changed over the past decade. In particular, since Ben Bernanke serves as the chairman, the Federal Reserve System provides more information, and provides it earlier to the public than ever before in history. To give an example, the federal open market committee of the Fed recently decided to increase the frequency of their economic projections and balance of risks statements from a biannual to a quarterly basis. The natural question that arises is whether these recent developments are necessarily good or whether there are still reasons for central banks not to share their insights and plans with the public. To address this question we develop a model that incorporates aggregate risk, e.g. random changes in GDP, and idiosyncratic risk that captures the random dispersion of income across households. The latter type of risk is – at least in parts – insurable in contracts consistent with voluntary participation incentives, or equivalently, contracts in a situation without commitment. That means that the exchange of goods prescribed by the contract must be such neither party under any circumstances has an incentive to renege. We model communication policies of policy makers as being concerned with the aggregate non-insurable risk, and assume that political decisionmakers have the possibility to publicly announce information on this type of risks early. We embed this specification of risk and information into a monetary production economy and assess the social value of policy announcements.

The second question – which I try to answer (jointly with Paustian Matthias) in chapter 3 – is how monetary policy should be conducted in the presence of conflicting policy aims in a stochastic general equilibrium model. The conflict in goals stems from the presence of two frictions that cannot be offset simultaneously if the nominal interest rate is the only policy instrument. The first friction is a transaction friction that results in a demand for cash. In order to equate the private opportunity costs for holding money to the zero social costs to produce it, the nominal interest rate should be zero. The second friction are nominal rigidities or price sticky prices. Unless, the nominal interest rate corresponds to the real interest rate, relative prices are distorted and resources are inefficiently used. However, in this case money holdings are costly. Analyzing optimal monetary policy in the short and in the long run we find that the resolution of this trade-off depends not only on the frequency of price adjustment – an argument which is stressed in the literature (e.g. Schmitt-Grohé and Uribe 2004, 2007; Woodford 2003). In addition, the interest elasticity of money demand plays a key role as the relevant determinant for the welfare costs of positive nominal interest rates. A paper version of this chapter has recently been accepted for publication at the *Journal of Economic Dynamics and Control* (Paustian and Stoltenberg 2008).

The third topic I examine is how monetary policy should be conducted to prevent unnecessary fluctuations in macroeconomic aggregates. To be more specific I ask whether the conditions for local stability and uniqueness of equilibrium sequences and the determination of the absolute price level are affected by the existence and timing of real balance effects as suggested by Patinkin (1949, 1965). The analysis is set in a dynamic general equilibrium model with flexible prices. Real balance effects are introduced with the money-in-the-utility-function specification (Sidrauski 1967), where consumption and money balances enter in a non-separate way such that this approach is equivalent to a shopping-time or real-resource costs-of-transactions specification. I employ two different assumptions on the stock of money that is assumed to deliver transaction services, either the one at the beginning or at the end of the period. While the former can be interpreted as a timing of markets, where the goods market is closed before the asset market is opened, the timing of markets is reversed if end-of-period money yields transaction services. Following the tradition of Sargent and Wallace (1975), I investigate for the different timing conventions whether the implementation of monetary policy with a nominal interest rate rule or a money growth rule plays a role for the determination of the price level and for conditions that ensure macroeconomic stability.

Summing up, the topics of my thesis fall into three popular fields in dynamic macroeconomics and monetary economics: efficient risk-sharing arrangements and the social value of public information, optimal fiscal and monetary policy, and the interactions of monetary policy, real and nominal determinacy. In the following, I provide a brief review of the literature on these three issues. I then summarize the results of my thesis in relation to the previous work.

1.2 Literature review

In this section I overview the existing literature on efficient risk-sharing arrangements and on the social value of public information, optimal fiscal and monetary policy, and on the interactions of monetary policy, real and nominal determinacy. While the last two fields are closely connected, there are almost no intersections between the fields and the first theme. In general, for ease of exposition, I report on the main contributions to each topic in a chronological order.

1.2.1 Efficient risk-sharing arrangements and the social value of public information

The literature that I review in this subsection relates to the chapter 2 of my thesis. The two strands of literature that I cover here, the literature on efficient risk-sharing and on global games are quite distinct, and thus they are treated separately.

Efficient risk-sharing arrangements

Hirshleifer (1971) was among the first to note that public information on insurable risks may make agents worth off from an ex-ante perspective. He considers the following example. There are two states of the world, agents are risk averse and the distribution of income differs across the agents. The random dispersion of income constitutes the idiosyncratic risk that agents are willing to share. If agents can trade the risk in complete asset markets before uncertainty about the state of nature is realized, they will share the risk. Intuitively, if the agents are aware of the state before they have the possibility to trade, there will be no trade at all, and each agent consumes his endowment. From an ex ante perspective this is inferior to an allocation of risk when the uncertainty is not resolved.

Schlee (2001) generalizes the result of Hirshleifer and states conditions under which all agents in a competitive equilibrium are worth off by an increase in precision of public information on tradable risks. In particular, he shows this is true in the absence of aggregate, non-insurable risk or when some agents are risk neutral. Even more important, better information is undesirable if the economy's aggregate demand for consumption has the representative agent property. That means that the aggregate demand function does depend on aggregate wealth but not on the particular distribution of wealth across agents. The latter result is surprising and points to an inconsistency of the representative agent approach: the representative agent consumes all the wealth and is therefore indifferent to information. This implies that starting with a representative agent is Pareto-inconsistent with the preferences of the agents of an economy with heterogenous endowments though the economy features the representative agent property in the sense described above.

In general, the precision of information must not be exogenous. Thus a natural question that arises is what happens to the Hirshleifer result if the arrival of public information on the insurable risk is endogenous. Berk and Uhlig (1993) study this question in a multiperiod model of asset markets which is dynamically complete if no additional information is released at intermediate nodes. Dynamic completeness means that though there does not exist enough long-lived state-contingent securities to span all possible nodes in the final period, the number of securities may be sufficient if agents are allowed to trade not only in the initial period but in intermediate periods, too (e.g. Duffie and Huang 1985; Kreps 1982). Berk and Uhlig show that an economy with dynamically complete markets may convert into an incomplete markets economy if the arrival of information on idiosyncratic risks is endogenous. If at least one agent achieves a higher utility in the incomplete markets equilibrium, and heterogeneity in preferences across agents is large enough, the agent is willing to bear the costs for releasing information early. After uncertainty is resolved, the idiosyncratic risk cannot be traded anymore at intermediate nodes rendering the economy dynamically incomplete.

One observation from individual consumption data is, that – for given per capita consumption – individual consumption is not only positively correlated with the average income but with current individual income, too (e.g. Cochrane 1993; Townsend 1994). Assuming a frictionless economy, this fact indicates an inefficient allocation of risk across agents. To put it differently, agents share only partially their idiosyncratic income risk. A growing literature argues this result can be an optimal individual consumption response in the absence of a commitment technology to induce enforcement of risk-sharing arrangements between agents (e.g. Coate and Ravallion 1993; Kocherlakota 1996a; Ligon, Thomas, and Worrall 2002; Thomas and Worrall 1988; Townsend 1994).

Focusing on long-term wage contracts as an example, Thomas and Worrall (1988) were among the first to analyze self-enforcing contracts or contracts consistent with voluntary participation incentives, i.e. contracts in which neither party has an incentive to renege. A long-run wage contract specifies a wage at each node, contingent on current and past wages. They find that wages in the optimal contract as opposed to spot market wages, which serve as the outside option, are sticky and less variable.

Coate and Ravallion (1993) characterize the best arrangement between two individuals that are subject to idiosyncratic risk as a non-cooperative equilibrium outcome in a simple repeated game. Restricting the attention to stationary, 'pure insurance' contracts, i.e. contracts that are contingent on current but not on past income realizations, they examine how the outcomes relate top the first best. They emphasize the relevance of this approach to understand consumption patterns in agrarian economies that often lack a legal and financial system to enforce contracts.

Kocherlakota (1996a) studies optimal consumption allocations as subgame-perfect allocations of a repeated game or equivalently as outcomes of an optimal consumption contract under two-sided lack of commitment, when information is symmetric. The idea is that under lack of commitment the exchange of goods proposed by the contract must be such that at any node agents at least weakly prefer to keep the agreement than defecting to autarchy as their outside option. He shows that if the first-best allocation is not in the set of sustainable allocations, current optimal consumption is indeed correlated with current income. In that sense, constrained-efficient allocations are consistent with consumption data. Furthermore, he proposes a sufficient statistic to discriminate in the data between allocations resulting from two-sided lack of commitment and symmetric information, and allocations that result under asymmetric information and monitoring costs as an alternative explanation (e.g. Atkeson and E. 1992; Phelan and Townsend 1991; Wang 1995).

The social value of information in global games

Should central banks announce precisely their future policies and economic forecasts to the public? This question is probably one of the most heavily discussed topics in media and in the literature on global games (e.g. Angeletos and Pavan 2007; Morris and Shin 2002; Morris, Shin, and Tong 2006; Svensson 2006; Woodford 2005). In particular in the light of Federal Reserve's shift in disclosure policy towards more transparency under chairman Ben Bernanke, this dispute is an active issue.

Morris and Shin (2002) examine the welfare effects of enhanced dissemination of public information in a 'beauty contest' model, in which agents decide on their optimal actions based on private and public information on the underlying fundamental state. In their setup agents face a strong coordination motive stemming from strategic complementarities in their actions. In their economy agents care about two things, choosing an action that is appropriate given the state of the economy and opting for an action that is sufficiently close to what the other agents are doing. To give an example, suppose that there is a large number of investors, that have to decide how much to invest in a variety of stocks. The return of stocks is linked to monetary policy by no-arbitrage conditions. Intuitively, the profits of the investors, depend on both, on investors' private information on future monetary policy and on public announcements of central banks. Since the return of stocks hinges on specific characteristics as well as on the development of the economy as a whole, investors have a motive to coordinate their actions and beliefs employing the public information provided by policy makers as a focal point.

Remarkably, Morris and Shin assume quadratic preferences such that individual preferences but not social welfare depend on the coordination motive. As their main result they find that an increase in precision of public information may be harmful to welfare if the coordination motive is strong enough and public information significantly less precise than private information. While an increase in the precision of the private information increases social welfare, the effect of an increase in the precision of the public signal is ambiguous. On the one hand, it will increase the accuracy of each market participant's assessment of the current state of the economy, with the result that equilibrium actions are on average more appropriate to current fundamentals. But on the other hand, it will reduce the weight that each market participant puts on her private information in forming her estimate of current conditions and hence in choosing her action, and increase the weight placed on the public signal instead. The latter effect results in an average action that is less appropriate to the current state: While the errors in participants' private signals cancel out on average, the error in the public signal affects everyone's action. This effect may outweigh the first one when agents wish to coordinate is sufficiently strong and private information is more precise than public information. Fueled by the socially undesirable coordination motive, agents place an inefficiently high weight on the relatively imprecise signal, instead of reacting to the more reliable private information, and the increase in volatility leads to welfare losses in the aggregate. However, in the absence of a signal-extraction problem – when public information is the only source of information – an increase in precision is always desirable.

Woodford (2005) and Svensson (2006) criticize the Morris-Shin result from a normative perspective and doubt whether the particular conditions necessary for a negative social value of information are of any practical relevance for central banks. In particular, Woodford's main criticism is that the strong coordination incentive necessary to render the value of public information negative, is ad odds with the type of preferences typically assumed in macroeconomic modeling. Moreover, he points out that the Morris-Shin result hinges crucially on the assumption that individual preferences but not social welfare feature the coordination motive. Svensson finds that the negative value of information requires a degree of precision in private information that is much superior to the one under public information: the former precision must be at least 8 times as great as the latter. This is not obvious if one thinks of the fundamental aggregate state as partially influenced by future policy actions.

Angeletos and Pavan (2007) develop a general framework to analyze equilibrium and efficiency of information in the presence of a signal extraction problem with strategic complementarity or substitutability. As an efficiency criterion they employ the notion of a decentralized command optimum. This optimum can be interpreted as the solution to a social planner's problem that maximizes ex-ante welfare, where the planner can perfectly control agents' incentives and their use of information, but cannot affect the precision of information. Referring to this benchmark, they classify economies by how the equilibrium use of information resulting form coordination or substitution relates to the efficient use of it. In particular, they clarify the Morris-Shin result and show that an increase in precision of public information can be detrimental to welfare only if the degree of coordination is higher than the socially optimal one.

In chapter 2 of my thesis, which is joint work with Vadym Lepetyuk, we analyze the social value of information in an environment where agents face aggregate risk - which is not insurable between agents - and idiosyncratic risk. In our specification households partially insure the idiosyncratic risk in contracts consistent with voluntary participation incentives. We assume that the policy maker learns about the aggregate shock before it directly impacts on the allocation, and can decide to provide that information to the public with certain precision.

1.2.2 Optimal fiscal and monetary policy

Chari and Kehoe (1999) provide an analysis of optimal fiscal and monetary policy in a dynamic stochastic general equilibrium model under flexible prices and perfect competition. In the absence of monetary policy, they study first how the government should set taxes on labor and capital income and the state-contingent interest on debt to finance a given stream of government expenditures. Throughout their analysis of Ramsey-optimal allocations with the primal approach they assume that the government does not have access to a non-distortionary lump-sum tax instrument. Their main lessons for policymaking reflect the Ramsey-taxation principle to smooth distortions over time and states of nature (Ramsey 1927). In particular, taxes on capital should be high initially and then zero (on average), holding taxes on labor constant and the state-contingent return is used to provide insurance against adverse shocks. Second, they introduce a transaction friction – either a cash-in-advance, money-in-the utility and shopping-time specification – and explore under which conditions the Friedman rule of a zero nominal interest rate is optimal. Remarkably, they find this to be the case under fairly general homotheticity and separability assumptions on preferences. Due to the optimality of a zero interest rate, inflation varies substantially over the business cycle, thereby absorbing the effects of government expenditure shocks.

A more recent literature studies Ramsey-optimal fiscal and monetary policy in environments with imperfect competition and nominal rigidities. In contrast to flexible prices the policy maker faces two distortions: price dispersion due to incomplete nominal adjustment calls for an optimal inflation of zero, implying costs of money holdings. However, the monetary distortion can only be offset by setting the nominal interest rate to zero. While Schmitt-Grohé and Uribe (2004) and Benignio and Woodford (2003) analyze the interaction of optimal fiscal and monetary policies, another part of the literature focuses on optimal monetary policy when the government has access to a lump-sum tax (Adão, Correia, and Teles 2003; Benignio and Woodford 2005; Khan, King, and Wolman 2003; Rotemberg and Woodford 1997; Woodford 2003).

Schmitt-Grohé and Uribe (2004) set up a stochastic production economy with sticky prices and a transaction friction but without capital. To finance an exogenous stream of expenditures, the government taxes labor income, issues debt or prints money. Schmitt-Grohé and Uribe assume real resource costs of transactions and calibrate their model to match stylized facts for the US economy. Due to the presence of nominal rigidities, they find that the optimal volatility of inflation is zero and that the Friedman is no longer optimal. The difference to Chari and Kehoe (1999) stems from the fact that under sticky prices, fluctuations in inflation induce distortions in relative prices leading to an inefficient allocation of resources. Correspondingly, the use of inflation as a shock absorber becomes less attractive when prices are imperfectly flexible, and the Ramsey planner relies on changes in the tax rates and in the level of public debt as a response to shocks.

While the Ramsey-approach is silent about the implementation of the optimal allocation, Woodford (2003) focuses on optimal monetary policies and their implementation in a linear-quadratic framework when prices are imperfectly flexible. He derives a quadratic strictly-microfounded policy objective as a second order approximation to representative household's utility and employs linear approximations to the conditions that describe the competitive equilibrium. Since it is assumed that the government finances her expenditures by a lump-sum tax the only policy instrument is the interest-rate operating target. Choosing price stability as the long run policy target (and approximation point), he shows that stabilization of the inflation rate is the main principle of optimal monetary policy in the short run – even in the presence of a transaction friction captured by a money-in-the-utility-function specification. As long as shocks affect the natural rate of interest (the nominal interest rate under flexible prices), Woodford points out that variations in interest rates are required to keep inflation stable. To put it differently: there is not only a well-known long-run conflict between offsetting the transaction and the pricing-friction but also a short-run trade-off in the optimal response to shocks. Bridging the gap to the practice of central banks, Woodford derives optimal linear interest-rate and inflation-targeting rules to implement the optimal allocation.

Adão et al. (2003) derive principals of optimal monetary policy in a stochastic maximizing economy, in which households'consumption expenditures are subject to a cash-advance constraint and firms set prices one period in advance. In contrast to previous contributions (Rotemberg and Woodford 1997; Woodford 2003) they find that the Friedman rule is optimal and show that the social planner under sticky prices can improve upon the optimal allocation under flexible prices if the planner has enough policy instruments. The logic of their argument goes as follows. If the nominal interest rate is zero the cash-in-advance constraint is not binding, and the level of real money balances is indeterminate. While there exists a unique equilibrium in consumption and labor under flexible prices, the allocation depends on the path of money supply when prices are sticky. Adão et al. show that the flexible-price allocation is included in the set of implementable allocations under imperfectly flexible prices. Correspondingly – depending on the specification of the fundamental shocks – the social planner is capable to achieve higher welfare if the policy maker has an additional policy instrument to pin down money supply and the optimal allocation, simultaneously.

Khan et al. (2003) provide an analysis how optimal policy should be conducted in the short and in the long run in the presence of nominal rigidities and a demand for cash modeled as a reduction in shopping-time when money is used in transactions. In contrast to Woodford (2003), Khan et al. (2003) treat the optimal deterministic steady state as the long run policy target as an integral part of the optimal policy problem that the Ramsey planner solves. Calibrating the model to match stylized facts of the US economy, they find that the Friedman rule is not optimal in the long run. Instead, the nominal interest rate should be positive but sufficiently low to allow for mild deflation. In the short run, they find support for the principles advocated by Woodford (2003): optimal monetary policy is characterized by price stability, thereby allowing for fluctuations in the interest-rate operating target.

Existing studies emphasize the frequency of price adjustments as determinant to quantify the welfare costs of nominal rigidities compared to utility losses stemming from a transaction friction. In chapter 3, which is joint work with Matthias Paustian, we show that the latter welfare costs crucially depend on the sensitivity of money demand with respect to changes in the nominal interest rate – a key fact which has been disregarded in the literature. Employing a standard money-in-the-utility-function specification, we show that this elasticity increases strongly as interest rates fall. In this environment we study optimal monetary policy in the short and in the long run when the nominal interest rate is the only available policy instrument.

1.2.3 Monetary policy, real and nominal determinacy

In their seminal paper Sargent and Wallace (1975) discuss the relationship between monetary policy and the determination of the absolute price level. In the context of a dynamic IS-LM model in which there is long-run neutrality of money and under rational expectations they compare the implications of alternative monetary policies. To be more precise, they compare how the implementation of monetary policy that either pegs the money growth rate or the nominal interest rate affects equilibrium determination. As their main result, they argue while the money growth rule leads to an uniquely determinate price level, the nominal interest rate peg leaves the absolute price level indeterminate (nominal indeterminacy). The rationale for this result is a homogeneity property of the economic system under interest rate policy: in the rational expectations equilibrium, the amount of real money balances but not prices and money separately are determined. This implies that there is an infinite number of equilibrium pairs for nominal money balances and the price level, and even equilibria may occur that are due to self-fulfilling revisions of people's expectations (sunspot equilibria). Assuming a welfare measure that punishes fluctuations in prices, Sargent and Wallace conclude with Friedman (1969) that monetary policy should rather target monetary aggregates than interest rates.

As a response, McCallum (1981) shows that the famous Sargent-Wallace indeterminacy arises only in case of purely exogenous interest-rate rules including contingency on the complete history of exogenous disturbances. However, if one allows for the possibility of an endogenous feedback – for example on the price level or inflation – the indeterminacy result is not inevitable.

In a series of papers (Leeper 1991; Sims 1994; Woodford 1994, 1995, 1996) highlight the role of fiscal policy for the determination of the absolute price level. Correspondingly, this line of research is often called the fiscal theory of the price level. The main result of this literature is that the Sargent-Wallace indeterminacy vanishes even under an interest rate peg if fiscal policy is specified in a way that breaks down the homogeneity property of the rational expectations equilibrium.

Leeper (1991) studies the local dynamics of interactions of fiscal and interest rate feedback rules in a stochastic maximizing endowment economy. If the fiscal authority is free to set her instrument without paying attention to the state of real government debt, fiscal policy is called 'active', 'passive' otherwise. A monetary policy is called 'active', if the central bank responds to a one percent increase in inflation with more than an one percent increase in the nominal interest rate. To put it differently, while an 'active' monetary policy increases the real interest rate, a 'passive' interest rate setting lowers it. Applying this classification, an interest rate peg is an example for a 'passive' monetary policy. Leeper shows that the absolute price level is determinate if at least one policy authority pursues an 'active' policy, otherwise the equilibrium is characterized by nominal indeterminacy. However, government's intertemporal solvency requires at least one of the policies to be set in a 'passive' way such that two 'active' policies violate the government budget constraint. In particular it follows that an interest rate peg uniquely determines inflation if fiscal policy is 'active'.

In a cash-in-advance model Woodford (1994) shows that while the stationary equilibrium associated with Friedman's rule of deflation yields the highest utility, implementing this policy by a constant money growth rate may be undesirable. The reason is that under the considered 'active' fiscal policy, the equilibrium under low constant money growth rates is prone to indeterminacy and the existence of sunspot equilibria. On the contrary, if the monetary authority implements Friedman's rule with an interest rate peg, the equilibrium is rendered unique. Key for these results is the type of fiscal policy under consideration: it involves no feedback on real government debt. This implies that in contrast to Sargent and Wallace (1975), the intertemporal government budget constraint adds to the set of equilibrium conditions which breaks down the homogeneity property of the rational expectations equilibrium.

Sims (1994) presents a representative-household endowment economy in which a demand for money stems from real resource costs of transactions. Sims extends and generalizes Woodford (1994) and Leeper (1991) in moving from local linear approximations techniques to the exact model solution. Validating Woodford (1994) globally, he finds that while a constant money growth rule may lead to an indeterminacy of the price level, an interest rate peg may not – depending on fiscal policy. In addition, he highlights that fiscal policy plays an important role for the determination of the price level, but the real allocation is in general independent of the fiscal policy regime as long as taxes are lump-sum.

Nowadays researchers mainly focus on policy regimes summarized by interest rate feedback rules, such as Taylor (1993), Benhabib, Schmitt-Grohé, and Uribe (2001a), Dupor (2001), Carlstrom and Fuerst (2001, 2005) or Woodford (2003). Correspondingly, the central problem of the theory of monetary policy is to provide guidelines how the central banks's interest-rate operating target should be set in response to various kinds of exogenous disturbances and endogenous variables.

Probably the most famous example of a proposal for setting the nominal interest rate is the one put forward by John Taylor (1993). The Taylor-rule says that the federal funds rate i_t is a linear function of the current inflation rate π_t and the output gap as the difference between log actual y_t and log potential real output y_t^p:

$$i_t = 0.04 + 1.5(\pi_t - 0.02) + 0.5(y_t - y_t^p), \tag{1.1}$$

where Taylor's numerical specification indicate an inflation target and a real federal funds rate of 2 percent on an annually basis. Taylor's proposal serves two purposes. On the one hand it provides a description of the actual policy of the U.S. Federal Reserve under chairman Alan Greenspan. On the other hand it is intended as a normative description how policy should be conducted. In particular, he emphasizes the importance to responding to a one percent increase of inflation above target with an increase of more than one percent in the policy instrument to ensure macroeconomic stability. The importance and robustness of an interest rate setting that is 'active' Leeper (1991) or consistent with the Taylor-principle, is a highly disputed topic in the literature on the conduct of monetary policy.

While Woodford (2003) and Galí (2008) emphasize the necessity of this principle for local stability and uniqueness of equilibrium sequences like inflation or output (real determinacy) in cashless economies, Benhabib et al. (2001a), and Carlstrom and Fuerst (2001) analyze conditions for real determinacy in economies that feature a demand for cash.

Benhabib et al. (2001a) analyze conditions under which interest-rate feedback rules that set the nominal interest rate as an increasing function of inflation lead to a determinate equilibrium. They consider two types of fiscal policies, Ricardian or non-Ricardian fiscal policies. Ricardian policies ensure that the present discounted value of total government liabilities converges to zero under all possible off-and on equilibrium paths of endogenous variables like the price level, inflation or the nominal interest rate.

Benhabib et al. emphasize that the determination of the price level crucially depends on the nature of fiscal policy but not on monetary policy: under non-Ricardian fiscal policies the price level is determined, while it is indeterminate under Ricardian policies. However, whether the real allocation is determined hinges on the stance of monetary policy and on the way in which money enters preferences and technology. In the context of a money-in-the utility-function model $u(c_t, m_t)$ with consumption and real money balances entering as Edgeworth-substitutes ($u_{cm} < 0$) and under flexible prices, they show that an active interest rate policy leads to multiple equilibria independent of the lump-sum fiscal policy regime. But if money is productive the Taylor-principle may lead to macroeconomic instability even in the more realistic case of Edgeworth-complements or real balance effects ($u_{cm} > 0$). Money is called productive if an increase in real money balances used in the production process leads to an increase in output available for consumption. Under these circumstances a 'passive' interest policy – responding to a one percent increase in the inflation measure with a less than one percent increase in the nominal interest rate – is necessary and sufficient for local stability and uniqueness of equilibrium sequences. When prices are imperfectly flexible, and fiscal policy is non-Ricardian or 'active', equilibrium sequences under a 'passive' interest rate setting converge asymptotically to the deterministic steady state. In contrast, if the central bank sets the interest-rate operating target according to the Taylor-principle multiple equilibria may arise.

Assuming intertemporal government solvency, Carlstrom and Fuerst (2001) study conditions for real determinacy of forward-looking Taylor-rules in a money-in-the-utility- function model under differing assumptions about which stock of money balances enters the utility function. The first timing convention is similar to a typical cash-in-advance timing. The amount of balances that yields utility is the one that households hold before entering goods trading but after adjusting their beginning-of-period money holdings through buying and selling bonds on the financial market. Alternatively, the authors assume that end-of-period money delivers utility. Their main results are as follows. Under cash-in-advance timing a necessary and sufficient condition for real determinacy is a 'passive' interest rate policy. However, if end-of-period money balances yield transaction services the equilibrium is determinate unless monetary policy responds with an increase of exactly one percent to an one percent increase in expected inflation above target.

Stressing the short-run perspective, existing studies on the conduct of monetary policy often ignore physical investment by assuming a fixed capital stock (Galí 2008; Walsh 2003a; Woodford 2003). In general, bonds and money are the only assets to transfer wealth across periods. Dupor (2001) adds physical investment to a standard continuous-time New Keynesian model and shows that an 'active' interest rate policy does not generate a locally unique equilibrium. On the contrary, a 'passive' monetary policy is necessary to bring about local equilibrium uniqueness.

Carlstrom and Fuerst (2005) analyze conditions for local determinacy in a New Keynesian model with physical investment, too, but in discrete-time. In contrast to Dupor (2001), they find support for the Taylor-principle as a stabilizing device even in the presence of capital accumulation. The reason for this result is that the no-arbitrage relationship between capital and bonds is different. In a discrete-time set up, the expected marginal productivity of capital equals the real interest rate; in continuous time, today's marginal productivity of capital equals the real interest rate. Exactly this difference alters the determinacy conditions across the two models.

The studies by Carlstrom and Fuerst (2001, 2005) and Dupor (2001) emphasize the importance of timing issues, and Benhabib et al. (2001a) highlight the role of preferences and technology for stability and uniqueness of equilibrium sequences. Combining these insights, I examine in chapter 4 of my thesis whether the existence and the timing of real balance effects affect conditions for local equilibrium uniqueness and stability of interest rate feedback rules. Throughout the chapter, I assume fiscal policies of the Ricardian type, implying that the intertemporal government budget constraint does not add to the set of relevant conditions in a competitive equilibrium, and fiscal policy is not capable to pursue policies that pin down the price level. In this context, I analyze whether the timing of real balance effects contribute to the determination of the absolute price level under interest policy as suggested by Patinkin (1949, 1965).

1.3 Outline of the thesis

In chapter 2 we analyze the welfare effects of public information provided by policy makers on future aggregate risks, including their future policies and economic forecasts. Agents in the economy face aggregate and idiosyncratic risk. The latter risk is partially insurable between households in self-enforcing contracts. The policy maker learns about the aggregate risk before it actually impacts on the allocation and has the option to announce this information to the public. We embed this specification of risk and information in a monetary production economy and examine whether central banks should perfectly or partially reveal their information on aggregate risks to the public.

In chapter 3 we address the question of optimal long run and short run monetary policy in the presence of conflicting policy aims. In particular, we consider an economy with nominal rigidities captured by firms setting prices in a staggered way, and a transaction friction modeled as a textbook money-in-the-utility-function specification. Throughout our analysis we abstract from interactions with fiscal policy and focus on the optimal setting of the interest-rate operating target as the only policy instrument. First, we determine the optimal deterministic steady state as the optimal long run policy target. Second, we approximate the model's structural equations around the optimal steady state and derive a purely quadratic approximation to households' ex-ante utility. This strictly-microfounded loss function serves as a policy objective for evaluating optimal monetary policy in the short run as optimal responses to various shocks that hit the economy.

In the last chapter of the thesis I examine the role of real balance effects for equilibrium determination. Monetary policy is either specified in terms of a current or forward-looking feedback rule for the nominal interest rate or as a peg for the growth rate of money supply. I examine how the determination of the absolute price level and the conditions for local stability and uniqueness of equilibrium sequences are affected by both – the implementation of monetary policy and preferences specifying the existence and timing of real balance effects.

2 Policy Announcements and Welfare

with Vadym Lepetyuk

*I*n the presence of idiosyncratic risk, the public revelation of information about uncertain aggregate outcomes such as policy choices can be detrimental to social welfare. By announcing informative signals on non-insurable aggregate risk, the policy maker distorts agents' insurance incentives and increases the riskiness of the optimal allocation that is feasible in self-enforceable arrangements. As an application, we consider a monetary authority that may reveal changes in the inflation target, and document that the negative effect of distorted insurance incentives can very well dominate conventional effects in favor for the release of better information.

2.1 Introduction

Nowadays central banks all over the world provide more information and release it earlier to the public than ever before in their history (Blinder, Ehrmann, Fratzscher, Haan, and Jansen 2008; Crowe and Meade 2008; Eijffinger and Geraats 2006; Woodford 2008). There seems to be widespread agreement that these recent changes in disclosure policies are socially beneficial. We argue that the case for disclosure is not that obvious. In particular, we show that by providing better information on future aggregate risk, e.g. by announcing future policies or revealing economic forecasts, policy makers may decrease social welfare by distorting private insurance incentives.

We consider an environment with idiosyncratic and aggregate risk. Households can voluntarily participate in insurance arrangements to reduce their consumption risk. Such arrangements are self-enforceable or compatible with voluntary participation incentives if in any period following the realization of idiosyncratic uncertainty, households choose not to walk away from the arrangement, and live in autarky from that period on. The latter option may be tempting for households with a high current income since the insurance arrangements prescribe transfers from these households to households with a low income in the current period. The lack of commitment thus creates a tension for high income households between higher current consumption and the future benefits of insurance promised in the arrangements.

Information plays a crucial role in households' trade-off between future insurance and current incentives. We study disclosure polices by introducing a public signal through which the future aggregate state is revealed. The signal is common to all agents and does not resolve households' idiosyncratic uncertainty. After the realization of current period idiosyncratic income and given the public signal on future aggregate risks agents decide to participate in social insurance.

As our main result, we formally show that less precise public information about the future aggregate state is preferable over perfect public information when incentive constraints matter. The mechanism is the following. Under the socially optimal insurance arrangement, the amount of the consumption good that the agents with high income in the current period are willing to transfer reflects future benefits of the insurance relative to the outside option. The key point is that agents value the insurance arrangement conditionally not only on their idiosyncratic realization but also on the signal about the aggregate state. In particular, if the signal indicates that the future aggregate state is likely to be one in which the benefits of the arrangement are relatively large, then the agents are willing to give up a larger share of current period consumption goods for these future benefits of the arrangement. Similarly, if the signal informs of a future aggregate state in which the gains of the risk-sharing agreement are relatively low, then agents with a high current income are less willing to share their good fortune. When the signal on the aggregate state becomes more informative, the optimal consumption allocation spreads out to account for all possible realizations of the signal. For high income agents, the expected utility before the signal materializes is the same under informative and uninformative signals. This implies that the consumption allocation of high income agents under perfect information is riskier than under imperfect information. Since households are risk averse, under perfect information high income agents are less willing to transfer goods to low income households. Correspondingly, under imperfect information low income households are better off, and from an ex-ante perspective agents prefer uninformative policy announcements.

Unlike Hirshleifer (1971) and his successors (Berk and Uhlig 1993; Schlee 2001), we focus on the welfare effects of more precise signals on aggregate, not on idiosyncratic risk. This difference is substantial: there are aggregate states in which more precise signals actually lead to better risk sharing, which cannot happen in case of signals on idiosyncratic risk. In these states, the value of the arrangement relative to the outside option is high, and thus better informed high income agents are willing to share more. Like in Hirshleifer, the overall effect of information is negative but relies here on the relevance of voluntary participation incentives for risk sharing. If agents were to respect commitments or trade a complete set of perfectly enforceable insurance contracts, better public information on aggregate risk would not affect social welfare.

To the best of our knowledge, we are the first to shed light on the welfare effects of announcements on risks that are common to all agents under the plausible assumption that the idiosyncratic risk is not completely, but only partially insurable.

As our main application, we develop a stochastic equilibrium model that integrates the risk-sharing mechanism into a monetary production economy in which households are subject to cash-in-advance constraints and face idiosyncratic employment opportunities. To insure against the idiosyncratic risk, households may engage in risk-sharing arrangements consistent with voluntary participation incentives. The monetary authority is assumed to pursue a stochastic inflation target. The target is known to the monetary authority one period in advance, and the authority may choose to release that information with certain precision. Our novel finding in this environment is that more precise announcements on future monetary policy are detrimental to social welfare. Furthermore, we show that the level of patience needed to sustain perfect risk sharing as the first best allocation is strictly increasing in the precision of the monetary policy announcement.

To evaluate the detrimental effect of policy announcements, we extend the model by introducing a fraction of firms, which need to set prices one period in advance. With this extension, better information affects the economy in two ways. First and conventionally, more precise announcements allow the sticky price firms to preset their prices more accurately, thereby resulting in less price distortions and a better allocation of resources. Second – and this is the new effect – early announcements distort risk sharing, increase consumption inequality and thereby worsen the contractual insurance possibilities ex-ante. We calibrate the monetary production economy to match basic inflation and income characteristics of the U.S. economy on an annual basis. The negative effect of information on aggregate risk is sizeable: the cost of information disclosure accounts for 18 percent of the benefit from removing aggregate fluctuations all together. Employing recent evidence on the frequency of price adjustments (Bils and Klenow 2004), the negative effect of information quantitatively dominates the positive aspect for reasonable degrees of risk aversion. Furthermore, the recent increase in income inequality in the U.S. (Gottschalk and Moffitt 2002; Krueger and Perri 2006) amplifies the negative rather than the positive effect of public information.

The social value of information has been extensively studied in the literature. Our paper builds a bridge between two distinct strands of literature: the literature on global games that focuses on aggregate risk, and the literature on efficient risk sharing that concentrates on the insurance of idiosyncratic risk. The model we develop puts us into the position to analyze the welfare effects of more precise information on the aggregate state of the economy under the realistic assumption that the idiosyncratic risk is not fully diversifiable. Moreover, the analysis of the welfare effects of better information on aggregate risk involves technical challenges that are absent in frameworks that focus on idiosyncratic risk.

In a global games framework, Morris and Shin (2002) show that better public information on aggregate risks may be undesirable in the presence of private information on these risks when the coordination of agents is driven by strategic complementarities in their actions. The result is due to the inefficiently high weight that agents assign to public information relative to private information. While the conditions for a welfare-decreasing effect of more precise public information are rather special and controversial (see e.g. Svensson 2006; Woodford 2005), Angeletos and Pavan (2007) draw a general conclusion that in the presence of a signal-extraction problem the social value of information is ambiguous if the first best is different from the equilibrium under perfect information. The main focus in this field is on aggregate risk, while idiosyncratic risk is either absent or assumed to be completely insurable due to the existence of complete financial markets.

Our study is closely related to the literature on efficient risk sharing. Hirshleifer (1971) is among the first to point out that perfect information makes risk averse agents ex-ante worse off if this leads to evaporation of risks that otherwise could have been shared in a competitive equilibrium. Schlee (2001) shows under which general conditions better public information about idiosyncratic risk is undesirable.

Kocherlakota (1996a) shows that the lack of commitment can explain the empirically observed positive correlation between current income and current consumption. The properties of stationary contracts in comparison to the first best are characterized by Coate and Ravallion (1993). Attanasio and Rios-Rull (2000) and Krueger and Perri (2005) argue that in economies where agreements are not enforceable, public insurance may crowd out private insurance arrangements. This literature focuses on the role of information on idiosyncratic risk in efficient risk-sharing arrangements. More relevant and important from a practical perspective, we consider the role of information on aggregate risk.

The remainder of the paper is organized as follows. In the next section, we start with a simple two-period example to highlight the basic voluntary risk-sharing mechanism involved, and state our main result in that simple environment. In Section 2.3 we set up a model that integrates the mechanism into a monetary production economy with infinite horizon and flexible prices. In Section 2.4 we state the main results for that application. In Section 2.5 we evaluate the importance of the distortions of risk-sharing possibilities caused by policy announcements. The last section concludes.

2.2 Simplified two-period real economy

We set up a simple example that captures the interaction between individual incentives for sharing idiosyncratic risk and the precision of public signals on aggregate risk. When participation in a risk-sharing arrangement is voluntary we show that risk averse agents prefer completely uninformative public signals on the aggregate risk over perfectly informative signals.

Consider a two period pure exchange economy with a continuum of ex-ante identical agents. In each period an agent obtains either a high endowment y^h or a low endowment y^l with equal probability – independent across time and agents. Furthermore, in the second period households' income is affected by taxes.[1] To ease the exposition, we assume that with equal probability the government can either tax away all goods (type-b policy) or impose zero tax (type-g policy), and assume that tax revenues are completely wasted by the government.

The preferences of agents are given by

$$E[u(c_1) + \beta u(c_2)], \tag{2.1}$$

where c_1 and c_2 are consumption in the first and in the second period respectively, β is the discount factor, and the period utility function $u(c)$ is increasing and strictly

[1] The tax is a convenient and general way to introduce aggregate risks associated with government policies. It also includes the inflation tax we consider in the next section.

concave. We measure social welfare according to (2.1), i.e. as households' expected utility before any uncertainty has been resolved.[2]

If agents are able to commit before their endowments realize in the first period, the optimal risk-sharing arrangement is perfect risk sharing. The commitment requirement is crucial. After observing current endowments an agent with a high income may have an incentive to deviate from the perfect risk-sharing agreement, making such an agreement unsustainable.

To capture this rational incentive we analyze risk-sharing possibilities under two-sided lack of commitment by introducing voluntary participation constraints. In the two-period model, the voluntary participation constraints apply only for the first period and characterize the trade-off between the first period consumption and the value of risk sharing provided by the arrangement in the second period. A risk-sharing arrangement is sustainable if each agent after observing his first period endowment at least weakly prefers to follow the arrangement than to defect into autarky. In other words, it is in the best rational interest of each agent to support the agreement. For the second period we assume that agents respect the commitments made in the first period. Otherwise, if voluntary participation were allowed in both periods, there would be no room for social insurance as agents would always choose to consume their endowments. While commitment for the second period is necessary for the existence of insurance in the two-period model, we do not need to impose any commitment in the infinite horizon model provided in the next section.

We compare two environments different in information precision about the future government policy. In the environment of perfect information agents know the second period government policy when in the first period they decide to sustain the risk-sharing agreement or to deviate to autarky. In the environment of completely imperfect information agents are left uninformed about the government policy.

In the first environment, when future government policy is known, participation constraints are given by

$$u(c_{1g}^{h}) + \beta\frac{1}{2}\left(u(c_{2g}^{hh}) + u(c_{2g}^{hl})\right) \geq u(y^{h}) + \beta\frac{1}{2}\left(u(y^{h}) + u(y^{l})\right) \tag{2.2}$$

$$u(c_{1b}^{h}) + \beta u(0) \geq u(y^{h}) + \beta u(0) \tag{2.3}$$

[2]We consider equal Pareto weights across ex-ante identical agents. If we were to allow for non-equal Pareto weights social welfare would still be higher under imperfect information than under perfect information about aggregate risk.

$$u(c_{1g}^{l}) + \beta\frac{1}{2}\left(u(c_{2g}^{lh}) + u(c_{2g}^{ll})\right) \geq u(y^{l}) + \beta\frac{1}{2}\left(u(y^{h}) + u(y^{l})\right) \tag{2.4}$$

$$u(c_{1b}^{l}) + \beta u(0) \geq u(y^{l}) + \beta u(0), \tag{2.5}$$

where c_{1k}^{i} is period-1 consumption of an agent with y^i first period endowment under type-k government policy, and c_{2k}^{ij} is period-2 consumption of an agent with y^i endowment in the first period and y^j endowment in the second period. In the constraints we explicitly substituted $c_{2b}^{ij} = 0$ for type-b policy. The first two constraints are relevant for agents with high first period income and the latter describe the incentives of agents with low first period income. The left hand side of each constraint constitutes expected utility of the arrangement, and the right hand side is the value of living in autarky as the outside option.

The resource feasibility constraints are

$$\frac{1}{2}\left(c_{1g}^{h} + c_{1g}^{l}\right) = \frac{1}{2}\left(c_{1b}^{h} + c_{1b}^{l}\right) = \frac{1}{4}\left(c_{2g}^{hh} + c_{2g}^{hl} + c_{2g}^{lh} + c_{2g}^{ll}\right) = \frac{1}{2}\left(y^{h} + y^{l}\right). \tag{2.6}$$

The optimal risk-sharing arrangement in the perfect information environment is a consumption allocation $\{c_{1k}^{i}, c_{2k}^{ij}\}$ that maximizes ex-ante utility (2.1) subject to participation constraints (2.2)-(2.5) and resource constraints (2.6).

The second environment is set to represent completely imperfect information. In the first period after observing their current endowments – without knowing the government policy in the second period – agents decide about participation in the risk-sharing agreement. Correspondingly, the voluntary participation constraints read

$$u(c_{1}^{h}) + \beta\frac{1}{4}\left(u(c_{2g}^{hh}) + u(c_{2g}^{hl}) + 2u(0)\right) \geq u(y^{h}) + \beta\frac{1}{4}\left(u(y^{h}) + u(y^{l}) + 2u(0)\right) \tag{2.7}$$

$$u(c_{1}^{l}) + \beta\frac{1}{4}\left(u(c_{2g}^{lh}) + u(c_{2g}^{ll}) + 2u(0)\right) \geq u(y^{l}) + \beta\frac{1}{4}\left(u(y^{h}) + u(y^{l}) + 2u(0)\right), \tag{2.8}$$

where c_{1}^{i} is period-1 consumption of an agent with y^i first period endowment, and resource feasibility requires

$$\frac{1}{2}\left(c_{1}^{h} + c_{1}^{l}\right) = \frac{1}{4}\left(c_{2g}^{hh} + c_{2g}^{hl} + c_{2g}^{lh} + c_{2g}^{ll}\right) = \frac{1}{2}\left(y^{h} + y^{l}\right). \tag{2.9}$$

The optimal risk-sharing arrangement under completely imperfect information is a consumption allocation $\{c_{1}^{i}, c_{2k}^{ij}\}$ that maximizes ex-ante utility (2.1) subject to participation constraints (2.7)-(2.8) and resource constraints (2.9).

Our goal is to highlight that information about aggregate risk can be harmful for social welfare since it distorts the insurance of idiosyncratic risk under voluntary participation. The result is formally stated in Theorem 2.1. The intuition is the following. From an ex-ante perspective, the agents desire to share their idiosyncratic endowment risk. The optimal insurance scheme prescribes transfers from high income agents to low income agents in all states. While agents with a low income are never worth-off in the agreement, for agents with a high income to live alternatively in autarky may be an attractive outside option. The better informed high income agents are about the future tax policy the less willing they are to transfer resources to the less fortunate agents.

Theorem 2.1. *Under completely imperfect information social welfare is strictly higher than under perfect information about future government policies.*

Proof. One can distinguish three cases depending on which participation constraints are binding. In the first case, all participation constraints for high endowment agents under perfect and imperfect information are binding. In the second case, only the participation constraints for high income agents under type-b policy are binding. In the third case, which is an intermediate case between the first two, for high income agents the participation constraints under type-b policy and imperfect information are binding.

In the first case, it follows from the optimal risk-sharing problem that consumption of the agents under type-g policy should be perfectly smoothed over time for both information environments. In the imperfect information environment this condition reads

$$c_{1g}^{h} = c_{2g}^{hh} = c_{2g}^{hl},$$

and similarly under imperfect information

$$c_{1}^{h} = c_{2g}^{hh} = c_{2g}^{hl}.$$

The algebraic details for this result are provided in the technical appendix. Under type-b policy, agents consume nothing in the second period, and we immediately obtain that in the perfect information environment $c_{1b}^{h} = y^{h}$ and $c_{1b}^{l} = y^{l}$. We thus compare the information environments in terms of the first period allocations. From the binding participation constraints (2.2), (2.3), and (2.7) it follows that the first period allocations under the two informational environments are characterized by the following inequalities $c_{1g}^{h} < c_{1}^{h} < c_{1b}^{h}$, which are further illustrated in Figure 2.1.

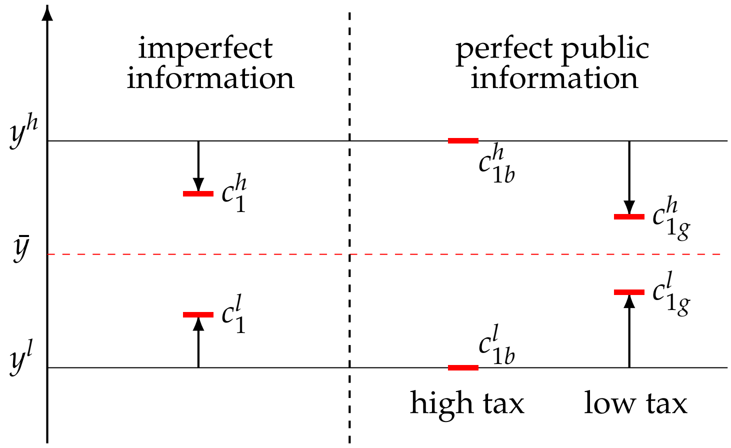

Figure 2.1: Optimal allocations for perfect and imperfect information under binding participation constraints.

From the binding participation constraints (2.2), (2.3), and (2.7) it also follows that agents with a high first period endowment obtain the same expected utility under perfect and imperfect information

$$\left(\frac{1}{2}+\frac{\beta}{2}\right)u(c_{1g}^h)+\frac{1}{2}u(c_{1b}^h)=\left(1+\frac{\beta}{2}\right)u(c_1^h). \tag{2.10}$$

Therefore the consumption allocation for the high income agents under perfect information is riskier from an ex-ante perspective. Due to strictly concave preferences, Equation (2.10) implies that

$$\left(\frac{1}{2}+\frac{\beta}{2}\right)c_{1g}^h+\frac{1}{2}c_{1b}^h>\left(1+\frac{\beta}{2}\right)c_1^h. \tag{2.11}$$

For the expected utility of agents with a low income in the first period under perfect and imperfect information this implies

$$\begin{aligned}
&\left(\frac{1}{2}+\frac{\beta}{2}\right)u(c_{1g}^l)+\frac{1}{2}u(c_{1b}^l)<\left(1+\frac{\beta}{2}\right)u\left(\frac{1+\beta}{2+\beta}c_{1g}^l+\frac{1}{2+\beta}c_{1b}^l\right) \\
&=\left(1+\frac{\beta}{2}\right)u\left(y^h+y^l-\frac{1+\beta}{2+\beta}c_{1g}^h-\frac{1}{2+\beta}c_{1b}^h\right) \\
&<\left(1+\frac{\beta}{2}\right)u(y^h+y^l-c_1^h)=\left(1+\frac{\beta}{2}\right)u(c_1^l),
\end{aligned} \tag{2.12}$$

where the first inequality is due to strict concavity and the second one is implied by (2.11). Thus, agents with low first period endowments are strictly better off under completely imperfect information. Taking unconditional expectation, adding up (2.10) and (2.12) we get that imperfect information is strictly preferable for this case.

In the second case when the participation constraints in the environment of imperfect information are not binding, the optimal allocation in this environment is perfect risk sharing. This outcome is preferable to the one under perfect information where the first best is not incentive compatible because the participation constraints for type-b policy (2.3) and (2.5) always hold with equality.

In the third case when the participation constraints for high first period endowment agents under type-g policy (2.2) are not binding but the participation constraints for high income agents in the completely uninformative environment (2.7) do bind, imperfect information is still preferable. It can be seen that as agents become more patient the first period allocation for perfect information cannot be improved upon, but under imperfect information social welfare is still increasing towards the first best. □

Compared to the literature on efficient risk sharing and public information (e.g. Berk and Uhlig 1993; Hirshleifer 1971; Schlee 2001), we show that not only public information on idiosyncratic risk but also on non-insurable aggregate risk can be harmful to social welfare. Unlike in that literature, there are aggregate states, in which perfectly informative signals improve risk sharing. This occurs when the government reveals type-g policy. Since the expected utility of the arrangement is high relative to the outside option, high income agents in this state are willing to share more with low income agents (see Figure 2.1).

The result of the negative social value of public information about the second period government policy is robust to any policies which lead to a non-identical dispersion of agents' disposable income. For example, if the tax were lump sum or if the government were to redistribute the tax revenues equally among agents, better information on the taxes would be still undesirable. Moreover, it is not crucial for the finding in Theorem 1 to require a policy under which the idiosyncratic risk vanishes completely. Even if taxation were not as extreme as a 100% tax, the result on the negative value of information stays valid.

Morris and Shin (2002) too provide an argument for a negative value of better information on aggregate risk in the presence of a signal-extraction problem. However, their argument has been criticized from a normative perspective (Woodford 2005). Woodford's main criticism is that the strong coordination incentive necessary to render the value of public information negative is at odds with the type of preferences typically assumed in macroeconomic modeling. Moreover, he points out that the Morris-Shin result hinges crucially on the assumption that individual preferences, but not social welfare feature the coordination motive. In contrast, we show that the social value of information can be negative even under standard preferences and even when individual preferences and social welfare coincide.

There are numerous possible applications including the welfare assessment of announcements on future tax, spending, debt or monetary policies, as well as the welfare effects of the public disclosure of economic forecasts. Because of its value for many economic decisions, even the general public pays special attention to information revealed by monetary authorities. Announcements by fiscal authorities on the other hand are less surprising since in developed countries fiscal decisions are mainly adopted through prolonged parliamentary mechanisms.

In the next section we therefore embed the risk-sharing mechanism into a richer environment with a monetary authority which announces a signal on its future inflation target. In that application we extend the simple example in several dimensions. First, we do not impose any commitment and consider an economy with an infinite number of periods. Second, we allow for continuity in information precision.

2.3 Monetary policy and infinite horizon

We proceed by integrating the voluntary risk-sharing mechanism into a monetary production economy. In this section we introduce an economy and describe the notion of equilibrium. In the economy, households' consumption expenditures are linked to nominal balances from the previous period with a cash-in-advance constraint originated by Clower (1967). As in Lucas (1980), each household consists of a worker-shopper pair. The production part comprises two sectors. Each sector is populated by a continuum of monopolistic competitive firms (Blanchard and Kiyotaki 1987; Dixit and Stiglitz 1977). Sectors differ in the productivity of the monopolistic firms. The random assignment of workers to firms with different productivity constitutes idiosyncratic risk. The notion of equilibrium is introduced in two steps. First, we define an equilibrium for given risk-sharing transfers among households. Second – and this is our main contribution here – we introduce the possibility for households to insure the idiosyncratic risk in arrangements that are consistent with their rational participation incentives (Kocherlakota 1996a). The exchange of consumption goods prescribed by the arrangements is reflected in risk-sharing transfers among households. Furthermore, we define the optimal pure insurance transfers under voluntary participation in order to find out how informative signals on future inflation affect the optimal insurance.

We consider an infinite-period production economy with a continuum of households of measure one and a single perishable consumption good.

Households are identical ex-ante, and their preferences over the stream of consumption are given by

$$E\left[\sum_{t=0}^{\infty}\beta^t u(c_t^i)\right], \tag{2.13}$$

where c_t^i is consumption of household i in period t, $0 < \beta < 1$ is the time discount factor, and $u(c)$ is the period utility function. We assume the period utility function to be twice-differentiable, increasing, and strictly concave.

Each household consists of two members: a shopper and a worker. Each period, the worker earns idiosyncratic income and inelastically supplies one unit of labor to one of the two production sectors, while the shopper buys consumption goods. Money is the only means for facilitating transactions and transferring wealth across periods. The period budget constraint of household i is

$$M_t^i + p_t c_t^i = M_{t-1}^i + p_t w_t^f + d_t + p_t \tau_t^i, \tag{2.14}$$

where M_t^i are nominal money holdings at the end of period t, d_t are shares of nominal profits of monopolistically competitive firms, τ_t^i are real transfers prescribed by a risk-sharing arrangement, w_t^f is the real wage in production sector f where the worker is employed, and p_t is the aggregate price level.

A shopper and a worker are distinguished by activities. In each period, while a worker works and earns money, a shopper exchanges the money earned by the worker in the previous period for consumption goods

$$p_t x_t^i = M_{t-1}^i, \tag{2.15}$$

where $x_t^i = c_t^i - \tau_t^i$ is the amount of the consumption good directly bought in the market.[3]

The production part of the economy is represented by two production sectors. Both sectors include a final good firm and a continuum of intermediate good firms. In each period the final good firms in both sectors produce an identical consumption good by aggregating over sector-specific differentiated intermediate goods. The intermediate goods are aggregated into the final good with a constant elasticity of

[3] Alternatively, the cash-in-advance constraints can be stated with inequalities. However, allowing for inequalities and therefore for self-insurance does not affect our main results. In Section 2.5, we conduct the latter exercise as a robustness check.

substitution

$$y_t^f = \left(\int_0^1 (y_t^{fj})^{1-\rho} dj \right)^{1/(1-\rho)}, \tag{2.16}$$

where y_t^f is the amount of the consumption good produced by the final good firm in sector$f$, y_t^{fj} is an intermediate good produced by differentiated good firm j in sector f, and ρ is the inverse of the elasticity of substitution between differentiated goods. The production technology of the differentiated good firms is given by

$$y_t^{fj} = a_t^f l_t^{fj}, \tag{2.17}$$

where l_t^{fj} is the labor input. The productivity of the differentiated good firms a_t^f is the same for all intermediate good firms within a production sector, but different across the sectors.

Acting under perfect competition, final good firms minimize costs by choosing the factor demand for each intermediate good to satisfy aggregate demand. The cost minimization problem is

$$\min \int p_t^{fj} y_t^{fj} dj \tag{2.18}$$

subject to the technology constraint (2.16), where p_t^{fj} is the price of the intermediate good j that the final good firm in sector f takes as given.

The intermediate good producers operate under monopolistic competition. A measure λ of monopolistically competitive firms maximize profits subject to the actual demand for their product. The profit maximization problem of the monopolistically competitive firms with flexible price-setting is

$$\max p_t^{fj} y_t^{fj} - p_t w_t^f l_t^{fj}, \tag{2.19}$$

given the demand of the final good firm and nominal sector wages, and subject to the production technology (2.17). The other $(1-\lambda)$ firms preset prices a period ahead based on a public signal on inflation by solving the expected profit maximization problem

$$\max E_{t-1}[p_t^{fj} y_t^{fj} - p_t w_t^f l_t^{fj} | s_{t-1}], \tag{2.20}$$

where s_{t-1} is the signal released in period $t-1$ about inflation target in period t.

In each period, a worker is randomly assigned either to be employed in the sector of high productivity a^h, or to work for firms with low productivity a^l. After selling the final goods to the shoppers, a worker obtains labor income and an equal share of profits of all monopolistically competitive firms.

Monetary policy is characterized by a stochastic inflation target. All agents in the economy are rational and know the stochastic properties of the inflation target process. In addition, the monetary authority knows the inflation target one period in advance, and provides a public signal on the future inflation target with a certain precision. The exogenous process for the gross inflation target π_j is given by an i.i.d process with two states of equal probability: high inflation π_h and low inflation π_l.[4] Similarly, the public signal on the next period inflation target takes two values, a high realization s_h and a low realization s_l. The precision of the public signal is given by $\kappa \equiv \text{Prob}[\pi_j|s_j]$, with $1/2 \leq \kappa \leq 1$.

The actual inflation coincides with the inflation target by appropriate money injections in all states. Since seigniorage is spent on government expenditures,[5] the government budget constraint reads

$$p_t g_t = M_t - M_{t-1}, \tag{2.21}$$

where g_t denotes real government expenditures, and M_t is the aggregate money supply.

Definition 2.2. *An incomplete markets equilibrium is an allocation $\{c_t^i, x_t^i, M_t^i, d_t^f, y_t^f, y_t^{fj}, M_t, g_t\}$ and a price system $\{p_t, p_t^{fj}, w_t^f\}$ such that given exogenous processes for the inflation target $\{\pi_t\}$, the public signal $\{s_t\}$, the assignments of households to production sectors $\{a_t^i\}$, and the risk-sharing transfers $\{\tau_t^i\}$, and initial conditions for the distribution of nominal money balances $\{M_{-1}^i\}$, and initial price setting of non-flexible price firms $\{p_0^{fj}\}$, the following conditions hold*

(i) for each household i given prices $\{p_t, w_t^f\}$ and profits $\{d_t^f\}$, the allocation $\{c_t^i, x_t^i, M_t^i\}$ maximizes household's utility (2.13) subject to the budget constraint (2.14) and the cash-in-advance constraint (2.15),

(ii) for each production sector f given prices $\{p_t, w_t^f\}$, the production allocation $\{y_t^f, y_t^{fj}\}$, prices $\{p_t^{fj}\}$ and profits $\{d_t^f\}$ solve the cost minimization problem of the final good firms (2.18), and the profit maximization problems of the differentiated good firms (2.19) and (2.20),

[4]The inflation target process and productivity are assumed to be non-degenerate $\pi_l < \pi_h$ and $a^l < a^h$.

[5]Alternatively, when seigniorage is equally distributed back to households our main results stated in Theorem 2.12 stay valid.

(iii) monetary injections are consistent with the inflation target

$$p_t = \pi_t p_{t-1},$$

(iv) the government budget constraint (2.21) is fulfilled, and

(v) markets clear

$$\int c_t^i di + g_t = \int y_t^f df, \quad \int M_t^i di = M_t, \quad \int l_t^{fj} dj = \frac{1}{2}.$$

We assume that the low realization of the inflation target is large enough to satisfy the resource feasibility with non-negative government expenditures. When we refer to social welfare derived from a certain allocation, we mean the ex-ante utility (2.13), which is evaluated before any uncertainty has been resolved.

The main element of our model is households' risk-sharing arrangement under voluntary participation. Without risk-sharing transfers the consumption allocation that results from the incomplete markets equilibrium is not efficient from an ex-ante perspective due to market incompleteness which prevents households from optimal borrowing and lending. However, the efficient use of a complete set of securities requires commitment or enforceability of the arrangements. In the absence of commitment the consumption allocation can still be improved by risk-sharing transfers consistent with voluntary participation incentives. We determine the socially optimal transfer scheme under voluntary participation in the incomplete markets equilibrium. Voluntary participation in social insurance provided by the risk-sharing transfers means that in each period households may decline the offered risk-sharing arrangement. In such a case they live forever in an economy with no transfers, consuming only the goods bought directly in the market.

With this mechanism we seek to capture financial market imperfections in an abstract way – either incompleteness of the financial markets themselves or private agents' limited access to it. When participation incentives matter, the resulting equilibrium consumption allocations share key properties with individual consumption patterns in the data (Krueger and Perri 2006). In particular, lack of commitment results in a positive correlation between current income and current consumption – a stylized fact that cannot be explained in models with complete financial markets (Kocherlakota 1996a).

The timing of events is illustrated in Figure 2.2. In each period, first, agents obtain a public signal on next period's inflation target and observe the current period infla-

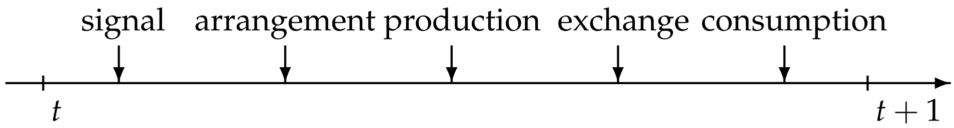

Figure 2.2: Timing of events in the monetary production economy.

tion target.[6] Second, households decide on sustaining a risk-sharing arrangement that prescribes transfers $\{\tau_t^i\}$. Third, workers and shoppers separate, and the former inelastically supply their labor services into the production process. Fourth, market exchange takes place. Flexible price monopolistic firms set prices for the current period, shoppers receive consumption goods in exchange for cash held from the previous period, workers receive wages and shares of profits and the government collects seigniorage from money injections. Fifth, among shoppers an exchange according to the risk-sharing arrangement takes place. Finally, members of each household meet again, consume, money balances are passed from the worker to the shopper for next period consumption purchases, and sticky price firms preset prices for the next period based on the public signal on the future inflation target.

Formally, the risk-sharing arrangement is built upon the consumption allocation of the incomplete markets equilibrium with no transfers as the outside option. This "off-equilibrium" allocation coincides with the equilibrium amount of consumption goods directly bought in the market $\{x_t^i\}$ since there is no choice how much money the agents hold from this period to next, and therefore how much they purchase. Moreover, since the equilibrium on the goods' market is not linked to the distribution of consumption among households, prices in the equilibrium without and with transfers are identical.

Let the individual public state at time t be $h_t^i = (x_t^i, X_t, s_t)$, where s_t is the public signal about inflation in period $t+1$, and X_t denotes aggregate resources available for private consumption. We restrict our analysis to pure insurance arrangements as emphasized by Kimball (1988), Coate and Ravallion (1993), and Ligon et al. (2002), which implies that the current risk-sharing transfers do not depend on transfers received in the past. Models that allow for history-dependent arrangements tend to overpredict the extent of risk sharing in practice (Alvarez and Jermann 2001; Krueger, Lustig, and Perri 2008). Tractability is an additional benefit. With pure insurance transfers we can analytically characterize the effect of information on social welfare.

[6] An alternative timing of events that leads to exactly the same results and does not require the awareness of current period inflation includes shoppers' trading first, followed by the risk sharing decision, and workers' realization of income.

Definition 2.3. *A consumption allocation $\{c_t^i\}$ is sustainable if there exist transfers $\{\tau_t^i\}$ such that*

(i) the consumption allocation $\{c_t^i\}$ solves the incomplete markets equilibrium with the transfers $\tau_t^i(h_t^i)$,

(ii) for each household i and state h_t^i, the consumption allocation $\{c_t^i\}$ is weakly preferable to the outside option $\{x_t^i\}$

$$E\left[\sum_{j=0}^{\infty}\beta^{t+j}u(c_{t+j}^i)|h_t^i\right] \geq E\left[\sum_{j=0}^{\infty}\beta^{t+j}u(x_{t+j}^i)|h_t^i\right], \tag{2.22}$$

(iii) and the transfers $\{\tau_t^i\}$ are resource-feasible

$$\int \tau_t^i(h_t^i)di = 0. \tag{2.23}$$

The key element of the information set in period t is the public signal on inflation provided by the monetary authority. The signal helps to resolve inflation uncertainty for the agents.

Definition 2.4. *A socially optimal arrangement under voluntary participation is a consumption allocation $\{c_t^i\}$ that provides the highest expected utility among the set of sustainable allocations.*

It is natural to compare the optimal arrangement under voluntary participation to an optimal arrangement under commitment. We define the *optimal commitment allocation* as a consumption allocation that provides the highest expected utility among the set of consumption-feasible allocations. An allocation is consumption-feasible if it solves the incomplete markets equilibrium with resource-feasible transfers $\{\tau_t^i\}$.

2.4 Negative social value of information

In this section we deliver our main result that policy announcements about future monetary policy can be detrimental to social welfare. We show that better precision of policy announcements is not desirable because it harms individual risk-sharing possibilities when rational participation incentives matter. In addition, we show that under more informative signals perfect risk sharing requires a higher degree of patience to be supported as a sustainable allocation.

To highlight the main effect we abstain in this section from the effect of public signals on optimal pricing decisions of firms. We avoid the pricing friction on the firm side by assuming in this section that all intermediate firms are flexible price firms. In the next section we extend the main result by illustrating a trade-off in public signal precision when a fraction of firms has to preset prices one period in advance: more precise information reduces the dispersion in relative prices between flexible and sticky-price firms and thereby leads to a better allocation of resources.

2.4.1 Optimal risk sharing under voluntary participation

In the following paragraphs we characterize the incomplete markets equilibrium under flexible prices, then proceed to state the problem to design the socially optimal arrangement in recursive form and derive general properties of the optimal solution.

As an initial point of our analysis we compute the incomplete markets equilibrium in the absence of transfers. Due to constant labor supply and since all firms are flexible in their price setting, the income of household i earned in period t depends only on worker's productivity in that period. From (2.16)-(2.19) the real income of a worker employed in sector f is equal to

$$w_t^f + \frac{d_t}{p_t} = \frac{1}{\mu}a^f + \frac{\mu - 1}{\mu}\frac{a^h + a^l}{2},$$

where $\mu = 1/(1-\rho)$ is a fixed mark-up above real marginal costs. The first term is labor income and the second term is profit equally distributed among households. From the cash-in-advance constraint (2.15), equilibrium consumption in the absence of transfers – the outside option – is given by

$$x_t^i = x^f(\pi_j) = \left[\frac{1}{\mu}a^f + \frac{\mu - 1}{\mu}\frac{a^h + a^l}{2}\right]/\pi_j, \tag{2.24}$$

when inflation in period t is π_j and the worker was assigned to sector f in period $t-1$. Combining the goods' market clearing condition with the government budget constraint (2.21) and the cash-in-advance constraint (2.15), government expenditures are

$$g_t = y_t - y_{t-1}/\pi_j = \frac{a^h + a^l}{2}\frac{\pi_j - 1}{\pi_j}. \tag{2.25}$$

It follows from (2.24) and (2.25) that the equilibrium consumption in the absence of transfers and the government expenditures is independent of the precision of the inflation target signal.

With risk-sharing transfers, from Definition 2.3 and Equation (2.24), period-t equilibrium consumption of household i is given by

$$c_t^i = c^f(\pi_j, s_k) = \left[\frac{1}{\mu}a^f + \frac{\mu-1}{\mu}\frac{a^h + a^l}{2}\right] / \pi_j + \tau(a^f, \pi_j, s_k),$$

when period-t signal of period-$t+1$ inflation is s_k, period-t inflation is π_j, and the worker of the household was assigned to production sector j in period $t-1$. With pure insurance transfers the equilibrium period-t consumption depends only on period-t direct purchases x_t^i, total resources available for private consumption X_t, and the signal s_t on the period $t+1$ inflation target realized in period t. In particular, this implies that the current transfers prescribed by the arrangement do not hinge on the individual transfers received in the past. This allows us to write the optimal risk-sharing arrangement problem in a recursive form.

For two inflation states and two signals on next period inflation rate the optimal contract problem given in Definitions 2.3 and 2.4 leads to the following recursive description

$$\max_{\{c^f(\pi_j,s_k)\geq 0\}} \frac{1}{1-\beta}V_{rs} \tag{2.26}$$

subject to participation constraints for high and low signals

$$\begin{aligned} &u(c^f(\pi_j, s_h)) + \beta\kappa V_{rs}(\pi_h) + \beta(1-\kappa)V_{rs}(\pi_l) + \frac{\beta^2}{1-\beta}V_{rs} \geq \\ &\qquad u(x^f(\pi_j)) + \beta\kappa V_{out}(\pi_h) + \beta(1-\kappa)V_{out}(\pi_l) + \frac{\beta^2}{1-\beta}V_{out} \quad \forall f, j, \end{aligned} \tag{2.27}$$

$$\begin{aligned} &u(c^f(\pi_j, s_l)) + \beta\kappa V_{rs}(\pi_l) + \beta(1-\kappa)V_{rs}(\pi_h) + \frac{\beta^2}{1-\beta}V_{rs} \geq \\ &\qquad u(x^f(\pi_j)) + \beta\kappa V_{out}(\pi_l) + \beta(1-\kappa)V_{out}(\pi_h) + \frac{\beta^2}{1-\beta}V_{out} \quad \forall f, j, \end{aligned} \tag{2.28}$$

and consumption-feasibility constraints

$$\sum_f c^f(\pi_j, s_h) = \sum_f c^f(\pi_j, s_l) = \sum_f x^f(\pi_j) \quad \forall j, \tag{2.29}$$

with the period values of the arrangement

$$V_{rs}(\pi_j) \equiv E\left[u(c^f(\pi_j, s_k))\middle|\, \pi_j\right], \qquad V_{rs} \equiv E\left[V_{rs}(\pi_j)\right],$$

and of the outside option

$$V_{out}(\pi_j) \equiv E\left[u(x^f(\pi_j))\middle|\, \pi_j\right], \qquad V_{out} \equiv E\left[V_{out}(\pi_j)\right].$$

As the first point in characterizing socially optimal arrangements, we show that the optimal arrangement exists and is unique.

Lemma 2.5. *The socially optimal arrangement exists and is unique. The arrangement and the social welfare are continuous functions in the precision of the public signal.*

The proof provided in the technical appendix employs the Theorem of the Maximum, and relies on the convexity of the set of allocations that satisfy participation constraints.

Next, we highlight some valuable characteristics of the optimal risk-sharing arrangement. Among the participation constraints (2.27) and (2.28) only restrictions for high productivity agents can potentially be binding for the optimal arrangement. Households assigned to low productivity firms are never worse off under the optimal arrangement relative to their outside option because the arrangement prescribes transfers from high productivity households as stated in the following lemma.

Lemma 2.6. *The socially optimal arrangement satisfies*

$$x^l(\pi_j, s_k) \le c^l(\pi_j, s_k) \le c^h(\pi_j, s_k) \le x^h(\pi_j, s_k).$$

The proof is provided in the technical appendix. First, we show that under the optimal arrangement in any state high income households consume at least as much as the low income households. Otherwise, if there are states such that low income households obtain more than the high income households, then an arrangement that prescribes perfect risk sharing in those states is sustainable and welfare improving. Second, we show that high income agents obtain not more than the outside option. By contradiction, either the participation constraint of some low productivity households is violated or a deviation can be constructed that yields higher social welfare.

As an immediate corollary from Lemma 2.6, the socially optimal arrangement satisfies $V_{rs}(\pi_j) - V_{out}(\pi_j) \ge 0$ for all inflation states π_j. In other words, in any inflation state the value of the optimal arrangement cannot be lower than the value of the allocation in the equilibrium without transfers.

2.4.2 Information, patience, and folk theorems

Before we proceed to our main result, we first pin down the cases when information precision does not affect social welfare, and then show that perfect risk sharing is less likely to be sustainable when the precision of public announcements increases. The following lemmas help to exclude these possibilities by characterizing the sustainability of the optimal commitment allocation and conditions when the outside option is the only sustainable allocation.

One potential candidate for the optimal risk-sharing arrangement is the optimal commitment allocation. Since all households are ax-ante the same, the optimal commitment allocation is perfect risk sharing $c_t^i = (x_t^h + x_t^l)/2$ for all households. Though voluntary participation imposes additional restrictions on the socially optimal arrangement, this does not mean that the optimal commitment allocation is never attainable. Indeed, perfect risk sharing may still be the socially optimal arrangement if the discount factor β is high enough. This result, commonly known as the folk theorem is established in the following lemma.

Lemma 2.7. *There exists a value $\bar{\beta}$ such that for any discount factor $\beta \geq \bar{\beta}$ the socially optimal arrangement for any signal precision is perfect risk sharing.*

Proof. Perfect risk sharing provides the highest ex-ante utility among the consumption-feasible allocations. The existence of $\bar{\beta}$ follows from monotonicity of participation constraints in β and $\bar{V}_{rs} > V_{out}$, where $\bar{V}_{rs}$ is the value of the perfect risk-sharing arrangement. In the participation constraints (2.27) and (2.28) a higher β increases the future value of perfect risk sharing relative to the allocation in the equilibrium without transfers, leaving the current incentives to deviate unaffected. Therefore, if the participation constraints are not binding for $\bar{\beta}$, they are not binding for any $\beta \geq \bar{\beta}$. □

On the lower end of sustainable arrangements, if the level of patience is relatively low, the set of sustainable allocations may shrink to one point, which is the equilibrium allocation in the absence of transfers. If the equilibrium with no transfers is the only sustainable allocation for a certain level of patience then the socially optimal allocation is again the outside option if households are even less patient.

Lemma 2.8. *If for a certain discount factor $\underline{\beta}$ the equilibrium allocation in the absence of transfers is the socially optimal arrangement for any signal precision, then for any $\beta \leq \underline{\beta}$ the socially optimal arrangement is the equilibrium allocation in the absence of transfers.*

Proof. Assume that for some $\beta \leq \underline{\beta}$ there exists an optimal arrangement different from the equilibrium allocation with no transfers. The arrangement allocation is sustainable. By Lemma 2.6, the value of this arrangement is at least as high as the value of defecting into the outside option for any inflation state. Then for $\underline{\beta}$ the allocation is also sustainable since the value of the arrangement other than the outside option gets an even higher weight in the participation constraints. This contradicts that for $\underline{\beta}$ the optimal arrangement is the no-transfer equilibrium allocation. □

In order to characterize the amount of consumption that high productivity households are willing to share with low productivity households it is useful to distinguish two opposite effects. The first effect is due to the increase in disposable resources available for consumption and therefore we refer to it as the *wealth effect*. Under low inflation, the disposable resources are higher, which tends to scale up the value of the arrangement, the value of the outside option, and the gain of the arrangement relative to the allocation of the no-transfer equilibrium. The second effect is related to the benefits of insurance, and we name the effect the *risk aversion effect*. Under high inflation consumers' disposable resources are lower, but this may lead to even higher benefits of risk sharing relative to the outside option if households' risk aversion is high enough.

In general, the wealth and the risk aversion effects lead households to value insurance differently in different inflation states. However, there is the degenerate possibility that these two effects exactly offset each other. This is the case when the relative gain of the optimal arrangement $V_{rs}(\pi_j) - V_{out}(\pi_j)$ is the same for all inflation states π_j.[7] Throughout the following analysis we exclude this possibility. Instead, either the wealth effect dominates when $V_{rs}(\pi_l) - V_{out}(\pi_l) > V_{rs}(\pi_h) - V_{out}(\pi_h)$, or the risk aversion effect dominates when the inequality is reversed.

We can now analyze how informative policy announcements influence the outcome of the optimal insurance arrangement under voluntary participation. Signal precision plays an important role for the sustainability of perfect risk sharing. In the

[7] The relative gain of the insurance arrangement for homogenous preferences vanishes when the degree of homogeneity converges to zero. The risk aversion effect (the wealth effect) dominates for a degree of homogeneity smaller (larger) than zero.

following proposition we show that the level of patience that is needed to sustain perfect risk sharing increases in the precision of the signal.

Proposition 2.9. *Let $\bar{\beta}(\kappa)$ be the cutoff point such that for each $\beta \geq \bar{\beta}(\kappa)$ perfect risk sharing is the socially optimal arrangement. The cutoff point $\bar{\beta}(\kappa)$ is strictly increasing in the precision of the public signal.*

The proof is provided in Appendix 5.1.4. The cutoff point is determined by a participation constraint for high productivity households that imposes the tightest restriction. Which particular constraint is the tightest depends on the gains the perfect risk-sharing arrangement offers relative to the equilibrium in the absence of transfers as can be seen from (2.27) and (2.28). The gain can be higher either under low or under high inflation. This depends on whether the wealth or risk aversion effect is dominant. However, in both cases the tightest constraint imposes a stronger restriction under informative signals than under uninformative signals. Suppose without loss of generality that the risk aversion effect dominates, i.e. the perfect risk sharing arrangement provides higher value relative to the equilibrium allocation without transfers under high inflation than under low inflation. While for high productivity agents the current period loss of staying in the arrangement is independent of signal precision, under the low next period inflation signal the expected future gain of insurance is lower for informative signals than for uninformative signals. Therefore, the level of patience needed to sustain the perfect risk sharing allocation is higher under an informative signal.

2.4.3 Information and welfare under partial risk sharing

A number of studies indicate that the more realistic case is when risk sharing is neither perfect nor absent, but partial.[8] This case is analyzed below. We show that the transfers prescribed by the arrangement depend on signal precision, and the signal can shape the resulting consumption allocation significantly. As our main result, we provide conditions for social welfare to be decreasing in the precision of the public signal. We exclude the cases when the optimal arrangement is either perfect risk sharing or the outside option and signal precision does not directly affect the arrangement and social welfare. Lemmas 2.10 and 2.11 provide sufficient conditions for a socially optimal arrangement that is neither perfect risk sharing nor the outside option.

[8] See e.g. Townsend (1994) or more recently Ligon et al. (2002).

If perfect risk sharing is not sustainable, a number of participation constraints of high productivity agents are binding. Which constraints are binding depends on the current loss relative to the outside option and the future value of the arrangement. We focus on the case when all constraints are binding and state below sufficient conditions for this case to apply.

Lemma 2.10. *If all participation constraints for high productivity agents are violated under an arrangement that prescribes perfect risk sharing in all states then all the constraints are binding under the optimal arrangement.*

The proof of this lemma is provided in the technical appendix. First, under the conditions of the lemma, we show that for all states the optimal arrangement satisfies strict inequalities $c^l(\pi_j, s_k) < c^h(\pi_j, s_k)$. Second, by contradiction we show that a Lagrangian multiplier on any participation constraint of a high productivity agent cannot be zero, since otherwise the inequalities do not hold.

Binding participation constraints imply that perfect risk sharing is not optimal, however on the other hand, the optimal arrangement may be given by another extreme, which is the outside option. In the following lemma we provide conditions under which there exists a socially optimal arrangement different from the consumption allocation in the absence of transfers. In particular, we consider a situation when the signal is uninformative.

Lemma 2.11. *Consider the case of an uninformative public signal with all participation constraints for high productivity agents binding in the optimal arrangement. If and only if*

$$\frac{1}{2}\left(\frac{u'(x^l(\pi_h))}{u'(x^h(\pi_h))} + \frac{u'(x^l(\pi_l))}{u'(x^h(\pi_l))}\right) > \frac{2-\beta}{\beta}, \tag{2.30}$$

then the socially optimal arrangement is not the consumption allocation of the equilibrium in the absence of transfers.

The proof is provided in the technical appendix. Under binding participation, the optimal arrangement should necessary solve a fixed point problem in terms of the value of the risk-sharing arrangement. The outside option is always a solution to the fixed point problem. The condition stated in the lemma guarantees that for an uninformative signal there exists another solution to the fixed point problem, which is a sustainable arrangement and is strictly preferable to the outside option.

From the perspective of an agent with a high current period income, risk sharing in future periods is attractive if the agent values the future significantly enough and if the agent is subject to high enough consumption risk in the equilibrium without transfers. Both aspects are reflected in condition (2.30) of Lemma 2.11. Taking it to one extreme, if future consumption is worthless for agents (i.e. $\beta = 0$), then the outside option is the only sustainable arrangement. Therefore, the threshold for β implied by condition (2.30) is strictly positive. On the other hand, if the consumption risk in the equilibrium without transfers is significant, the marginal utility for consuming the low income relative to the high income, $u'(x^l(\pi_j))/u'(x^h(\pi_j))$, may become substantial, and thus the required level of patience for engaging in social insurance is low.

In the following theorem we establish our main result that social welfare is strictly decreasing in the precision of the public signal.

Theorem 2.12. *If all participation constraints for high productivity agents are binding and the equilibrium allocation in the absence of transfers is not the only sustainable arrangement, then social welfare is strictly decreasing in precision of the public signal on future inflation.*

The proof is provided in Appendix 5.1.7. For any two values of signal precision $\kappa_1 < \kappa_2$, we construct a consumption allocation for κ_1 based on the optimal allocation for κ_2 as follows. The allocation is constructed to satisfy participation constraints for κ_1 with equality while the value of the arrangement in future periods corresponds to the optimal arrangement for κ_2. We show that this allocation delivers strictly higher welfare than the optimal allocation for κ_2, and is also sustainable for signal precision κ_1. Thus, since the optimal allocation for κ_1 must be at least as good as the one constructed, welfare is strictly higher for lower signal precision.

The negative influence of informative signals on social welfare can be illustrated as follows. Assume that the risk aversion dominates the wealth effect. Suppose further the realized signal indicates that the next period inflation is more likely to be low. From the signal households infer that the future value of the arrangement relative to the outside option is lower, which is an unfavorable outcome for all households. Therefore the high productivity agents require higher current period consumption. In contrast, under the high inflation signal, which indicates a higher value of the arrangement relative to the outside option, the high productivity agents can be satisfied with lower current period consumption. Compared to uninformative signals, the consumption allocation prescribed by the optimal arrangement diverges as precision increases, i.e. the consumption allocation of high income agents becomes riskier ex-ante. Binding participation constraints imply that the expected utility of high income agents before the signal realization is independent of signal precision. Since households are risk averse, high income agents are less willing to share their good fortune with low income agents when information gets more precise. Correspondingly, from the resource constraint it follows that low income households are better off under imperfect information. Therefore, ex-ante risk averse agents prefer uninformative policy announcements.

The negative value of information does not depend on whether the wealth effect or the risk aversion effect is dominant. If the wealth effect dominates, the high productivity agents require lower current period consumption following a low inflation signal, and demand higher current period consumption following a high signal. Nonetheless, from an ex-ante perspective such divergences are still welfare decreasing for risk averse agents.

We prove that social welfare is strictly decreasing in precision when all participation constraints for high productivity agents are binding. This is a sufficient condition. Our numerical computations reveal that as long as perfect risk sharing is not sustainable for uninformative signals, social welfare is strictly decreasing in precision no matter how many constraints are binding at the optimal arrangement. Evidently, if perfect risk sharing is sustainable under uninformative signals but not under informative signals – which can occur since the minimum level of patience needed to sustain perfect risk sharing is increasing in precision (see Proposition 2.9) – less information is still preferable.

The strongest effect of information on welfare is observed – measured as the difference in social welfare between uninformative and perfectly informative signals – when all participation constraints for high productivity agents are binding. The effect is weaker when in some inflation states the optimal allocation exhibits perfect risk sharing, which is the case when participation constraints are not binding in those states. Intuitively, in such a case the influence of information on risk sharing is limited to states with binding constraints, and the overall effect on the consumption allocation is smaller.

The result in Theorem 2.12 is robust with respect to the value of the outside option. The assumption of agents living forever in the equilibrium without transfers when a given risk-sharing arrangement is declined constitutes a harsh penalty. The main result stays valid qualitatively if this assumption is relaxed. Suppose the penalty were weaker, for example, if agents were allowed to reengage in social insurance. Then under the optimal arrangement the high income agents would be less willing to share the risk with the low income agents. In this case, since the marginal utility of low income households is higher, public information plays an even more significant role than under harsher punishment.

In this section we have characterized how the precision of public signals on future inflation affects optimal insurance under voluntary participation when prices are flexible. If the optimal arrangement is partial risk sharing, the precision of the signal effectively influences the distribution of consumption in the risk-sharing arrangement. We show that higher precision in signals is socially undesirable because this decreases the willingness of high income households to transfer resources to less fortunate households. In addition, we find that the level of patience needed to sustain the perfect risk sharing allocation is strictly increasing in the precision of the signal. The reason for this is that the public information provided by the monetary authority does not help agents to make better decisions for the future. In the next section we extend our framework to allow for a beneficial role of public information, and thereby assess the importance of the detrimental effect of policy announcements on risk sharing.

2.5 Assessment of risk-sharing distortions

The main purpose of this section is to evaluate the risk-sharing effect. To serve this goal, we introduce a positive effect of information by considering imperfectly flexible prices. We assume that a positive fraction of intermediate good producers preset their prices one period in advance (Woodford 2003), which results in increasing aggregate resources with better public information. We proceed to quantitatively assess the importance of the negative and positive effects of information by setting up a numerical example that shares some salient features with the U.S. economy. We find that the negative effect of information prevails for reasonable degrees of risk aversion. Furthermore, the increase in the U.S. income inequality over the last decades tends to amplify the negative role of public information about aggregate risk on social welfare. As robustness checks, we subsequently allow for a weaker penalty for default, for self-insurance, and staggered-price setting (Calvo 1983). As an alternative possibility to measure the negative effect of information and independent of a particular positive effect of information, we then compare the gain of uninformative signals relative to the elimination of all inflation fluctuations (Lucas 2003).

2.5.1 Imperfectly flexible prices

When some monopolistically competitive firms have to preset prices, firms' problems become non-trivial. Solving first the cost minimization problem of the perfectly competitive final good firms (2.18) we get the demand for each of the variety goods

$$y_t^{fj} = \left(\frac{p_t^{fj}}{p_t}\right)^{-1/\rho} y_t^f, \tag{2.31}$$

where the aggregate price level is defined by

$$p_t = \left(\int_0^1 (p_t^{fj})^{1-1/\rho} dj\right)^{1/(1-1/\rho)}. \tag{2.32}$$

Using the production technology (2.17), the final good firm demand (2.31), and integrating over all monopolistically competitive firms within a sector, production per worker in sector f is given by

$$y_t^f = \frac{a^f}{\Delta_t^f}, \tag{2.33}$$

where price dispersion $\Delta_t^f \equiv \int \left(\frac{p_t^{fj}}{p_t}\right)^{-1/\rho} dj$ satisfies $\Delta_t^f \geq 1$ by Jensen's inequality. The highest level of production is achieved when all differentiated goods firms are flexible in their pricing decision and, therefore, set the same price $p_t^{fj} = p_t$.

Signal precision under imperfectly flexible prices affects the outcome of the optimal insurance arrangement in two different ways. First, it influences the willingness of high productivity households to share with low productivity households, as highlighted in the previous section. Second, it affects the amount of resources that can be shared among the households. The influence of the latter effect can be illustrated by a particular participation constraint. With a fraction of prices preset and for price dispersion, which is symmetric in predicted and realized inflation,[9] the constraint in the recursive formulation for a high inflation signal (2.27) is modified to

$$\begin{aligned} &u(c^f(\Delta_{-1}^f, \pi_j, s_h)) + \beta\kappa V_{rs}(\Delta^f, \pi_h) + \beta(1-\kappa) V_{rs}(\Delta^f, \pi_l) + \frac{\beta^2}{1-\beta} V_{rs} \geq \\ &\qquad u(x^f(\Delta_{-1}^f, \pi_j)) + \beta\kappa V_{out}(\Delta^f, \pi_h) + \beta(1-\kappa) V_{out}(\Delta^f, \pi_l) + \frac{\beta^2}{1-\beta} V_{out}, \end{aligned} \tag{2.34}$$

[9] Symmetry implies that price dispersion for any signal realization depends on the precision of the signal but not on the signal itself.

where Δ^f and Δ^f_{-1} are the current and the previous period price dispersions, π_j is the current period inflation, and V_{rs} and V_{out} are the value of the arrangement and the value of the outside option defined accordingly. An increase in precision distorts risk-sharing opportunities when risk sharing is partial, but on the other hand it allows sticky price firms to set their prices more accurately, thereby resulting in less price distortions and a better allocation of resources. Taking it to the extreme, if the socially optimal arrangement is either the outside option or perfect risk sharing, then the expected utility of households is increasing in signal precision.

We compute social welfare in two steps. First, for any given precision we calculate prices and production by solving the problems of final good firms (2.18), and monopolistically competitive firms (2.19) and (2.20). Second, taking the resources available for consumption as given, we derive the value of the outside option and compute the optimal consumption allocation according to (2.26)-(2.29).

2.5.2 Quantitative assessment: imperfectly flexible prices

We set up a numerical example to assess quantitatively the effect of public announcements. The baseline is constructed to match stylized facts for the U.S. economy on an annual basis. We calibrate the inflation process to the postwar U.S. consumer price index that results in two states with 1.2 and 5.7 percent inflation rates. We set the variance of the productivity process σ_y^2 to 0.1, which is the average of the variance for the transitory component of within-groups income for the U.S. between 1980 and 2003 as estimated by Krueger and Perri (2006).[10] Throughout the example we employ standard preferences that feature constant relative risk aversion, and calibrate the elasticity of substitution between differentiated goods to a value of 6 following Woodford (2003). The fraction of sticky price firms is set to 13 percent, which is the value found by Bils and Klenow (2004) using U.S. data for 1995-1997 collected by the Bureau of Labor Statistics. We keep the discount factor at the highest value such that all participation constraints are violated under perfect risk sharing (the condition of Lemma 2.10) for any precision.

We measure the social value of policy announcements as the percentage difference in certainty equivalent consumption between uninformative and perfectly informative signals. In other words, this measure captures the percentage amount of annual consumption agents are willing to give up until they are indifferent between perfectly informative announcements and no announcements at all.

[10] Violante (2002) provides similar numbers for wage inequality.

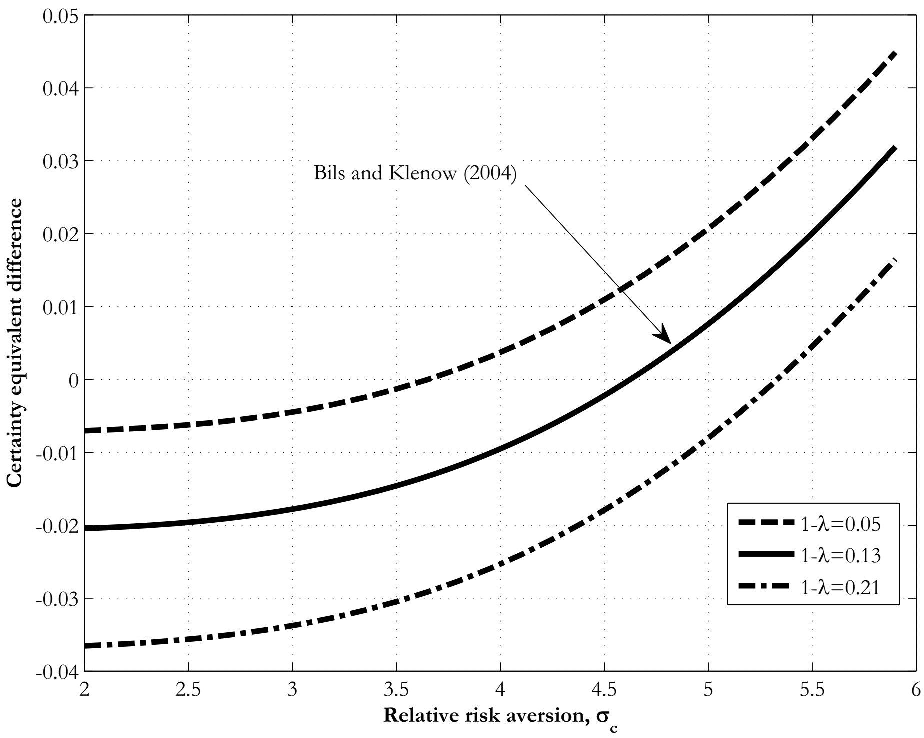

Figure 2.3: The welfare gain of uninformative signals relative to perfectly informative signals expressed in percentage certainty equivalent consumption as a function of risk aversion.

We find that the optimal announcements are either no announcement ($\kappa = 1/2$) or perfect announcements ($\kappa = 1$). The negative effect of information dominates for any coefficient of relative risk aversion that exceeds 4.66, which is not an unreasonably high value of the coefficient.[11] The result is illustrated in Figure 2.3 where the social value of information is shown as a function of risk aversion for three different fractions of preset prices, $1 - \lambda$, including 13%, which is our baseline value.

When a larger fraction of prices is adjusted more frequently the social value of information becomes negative for even lower degrees of risk aversion (see the dotted line for $1 - \lambda = 0.05$ in Figure 2.3). It is a well-documented fact the U.S. have experienced a substantial increase in income inequality over the last decades (see Gottschalk and Moffitt 2002; Krueger and Perri 2006). We capture this evidence by an increase in the variance of the income process σ_y^2 which results from the random assignment of workers to sectors of different productivity. How does this increase

[11]There is quite a controversy about the magnitude of the constant risk aversion coefficient (see Campbell 2003; Kocherlakota 1996b; Mehra and Prescott 1985). Kocherlakota (1996b) summarized the prevailing view "... that a vast majority of economists believe that values for [the coefficient of relative risk aversion] above ten (or, for that matter above five) imply highly implausible behavior on part of the individuals."

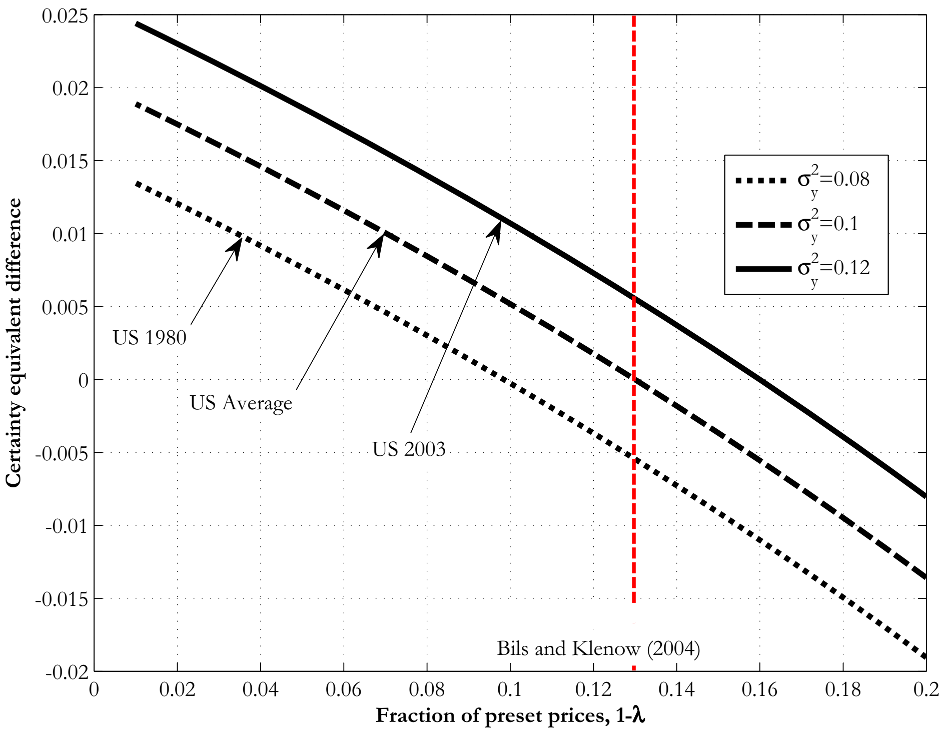

Figure 2.4: The welfare gain of uninformative signals relative to perfectly informative signals expressed in percentage certainty equivalent consumption as a function of the fraction of prices preset.

in income inequality affect the trade-off between the destruction of insurance possibilities on one hand and the better allocation of resources on the other hand, when policy announcements become more precise? For this exercise we set the coefficient of relative risk aversion to 4.66 – implying that the positive and negative effect of more precise information cancel out for the average of the idiosyncratic variance in the U.S. between 1980 and 2003. Employing our baseline calibration we obtain that for the variance of the idiosyncratic component of within-group income observed in 1980, the social value of information was positive. From 1980 to 2003 the variance increased from 8 percent to 12 percent (Krueger and Perri 2006). This renders the social value of information negative. This result is illustrated in Figure 2.4. For the income inequality observed in 2003, a secretive inflation target is desirable unless the fraction of prices preset for one year were exceeding 16 percent.

We proceed further by conducting three robustness checks: lower the penalty for default, allowing for self-insurance, and Calvo pricing.

Weaker penalty for default

The negative effect of information on social welfare is amplified when the penalty for default is decreased, i.e. the value of the outside option is higher. To illustrate this property, we compute the social value of information when households are allowed to reengage in social insurance after one period instead of living in the equilibrium without transfers forever. The corresponding participation constraint for a high inflation signal (2.34) is modified to

$$u(c^f(\Delta^f_{-1}, \pi_j, s_h)) + \beta\kappa V_{rs}(\Delta^f, \pi_h) + \beta(1-\kappa)V_{rs}(\Delta^f, \pi_l) \geq \\ u(x^f(\Delta^f_{-1}, \pi_j)) + \beta\kappa V_{out}(\Delta^f, \pi_h) + \beta(1-\kappa)V_{out}(\Delta^f, \pi_l).$$

Though qualitatively similar to our standard case in which agents are not allowed to reengage in risk sharing arrangements, the results differ quantitatively. Under a lower penalty for default, the negative aspect of information dominates the positive one for even lower degrees of risk aversion and even when idiosyncratic income uncertainty is lower (see Figures 2.5 and 2.6 in the technical appendix). For example, when the fraction of preset prices equals the value found by Bils and Klenow (2004), the negative effect of information outperforms the positive effect for degrees of risk aversion higher than 3.5. Moreover, even for an idiosyncratic income variance from 1980, the social value of policy announcements becomes negative in this scenario.

Self-insurance

Qualitatively similar results are obtained when we allow for the possibility of self-insurance captured by the cash-in-advance constraints written as inequalities, i.e. $p_t x^i_t \leq M^i_{t-1}$ instead of (2.15). This permits agents to save money for purchases in future periods. In our numerical example, agents nevertheless optimally choose not to save in the optimal arrangement. Self-insurance and voluntary transfers both facilitate consumption insurance, but self-insurance is associated with the burden of inflation costs, and therefore agents find it inferior.

The cash-in-advance constraints written with inequalities do however influence the optimal arrangement through the outside option. When deciding about participation in a risk-sharing arrangement, agents take into account that self-insurance increases the value of their outside option.[12] This implies that the high productivity agents have smaller incentives to share with the low productivity agents, and consequently, the optimal arrangement is worse from an ex-ante perspective. For our baseline calibration with flexible prices and coefficients of risk aversion from 1 to

[12]The value of the outside option is now the result of an optimization problem. To get accurate solutions for this optimal self-insurance problem we iterate on the value function subject to the cash-in-advance constraints formulated as inequalities.

4.66, the utility loss can add up to 3% measured in consumption equivalents. This implies a larger degree of consumption dispersion between high and low productivity agents, and the marginal gain of redistribution that can be achieved by uninformative signals is now higher than in the absence of self-insurance. As a result, the negative effect of information on social welfare is stronger, e.g. for a relative degree of risk aversion of 4, the welfare gain of uninformative signals in consumption equivalents is 0.024% with self-insurance as compared to 0.011% in the absence of it.

Staggered-price setting
Remarkably, the positive effect of information is mitigated under staggered price setting as in Calvo (1983), where each period firms face an invariant probability to reset their prices. Calvo firms weight over the current and an infinite number of future periods – the next one where the signal is informative and over the following periods in which they rely only on unconditional expectations. On the contrary, in our environment firms preset prices for one period only and thus put all the emphasize on that period for which the signal is informative. Correspondingly, all distortions in relative prices root in imperfect information on the aggregate state in that period. This in turn implies that the positive effect of information on aggregate resources is in general stronger when a fraction of firms has to preset prices one period ahead than under Calvo pricing.

2.5.3 Quantitative assessment: aggregate fluctuations

Instead of introducing a positive effect of information, the negative effect of policy announcements on future inflation targets can be evaluated relative to the well studied welfare gain from complete removal of aggregate fluctuations (Lucas 2003). For the baseline calibration with flexible prices, we calculate first the socially optimal arrangement with constant inflation equal to the average U.S. postwar consumer price inflation. Second, we compute social welfare with stochastic inflation under perfectly informative and under uninformative signals. A number of values for the coefficient of relative risk aversion are considered up to 50, which is the value implied by risk-premium puzzle (Mehra and Prescott 1985). We find that the welfare gain of completely uninformative signals relative to perfectly informative signals measured in certainty equivalent consumption is in the range of 9 to 53 percent of the welfare gain under a constant inflation rate (see Table 2.1). Even for reasonable degrees of relative risk aversion below five (see Kocherlakota 1996b), the relative

Coefficient of relative risk aversion, σ_c	2	5	10	20	50
Welfare gain, %	9.1	17.6	35.3	44.2	53.3

Table 2.1: Welfare gain of uninformative signals relative to perfectly informative signals as a percentage of welfare gain from inflation stabilization measured in certainty equivalent consumption.

welfare gain of uninformative signals on aggregate fluctuations accounts for up to approximately 18 percent, which can be referred to as sizeable.

2.6 Conclusion

In this paper we studied the welfare effects of policy announcements about future aggregate risk in the presence of idiosyncratic risk. We developed a stylized model of a monetary production economy that integrates optimal insurance arrangements for idiosyncratic risk under voluntary participation. In this environment, we analyzed how the precision of signals on future inflation targets affects social welfare.

The main message of the paper is that more precise announcements on future monetary policy can be detrimental to social welfare. By revealing information on future realizations of the aggregate risk, the policymaker distorts households' insurance incentives and thereby increases the riskiness of the optimal consumption allocation that is consistent with rational participation incentives.

The effect we describe is one additional channel through which public announcements affect social welfare. In the stylized model we show that the effect is sizeable relative to the commonly considered positive effects of information, and to the removal of aggregate fluctuations. While we focus on monetary policy announcements, the effect is relevant for any announcements of public policy. It should be taken into consideration by policymakers. The size of the effect for each particular application is subject of assessment and further research. This would require developing a fully-fledged stochastic general equilibrium model that comprises the insurance mechanism and is moreover able to match the salient facts of business cycles.

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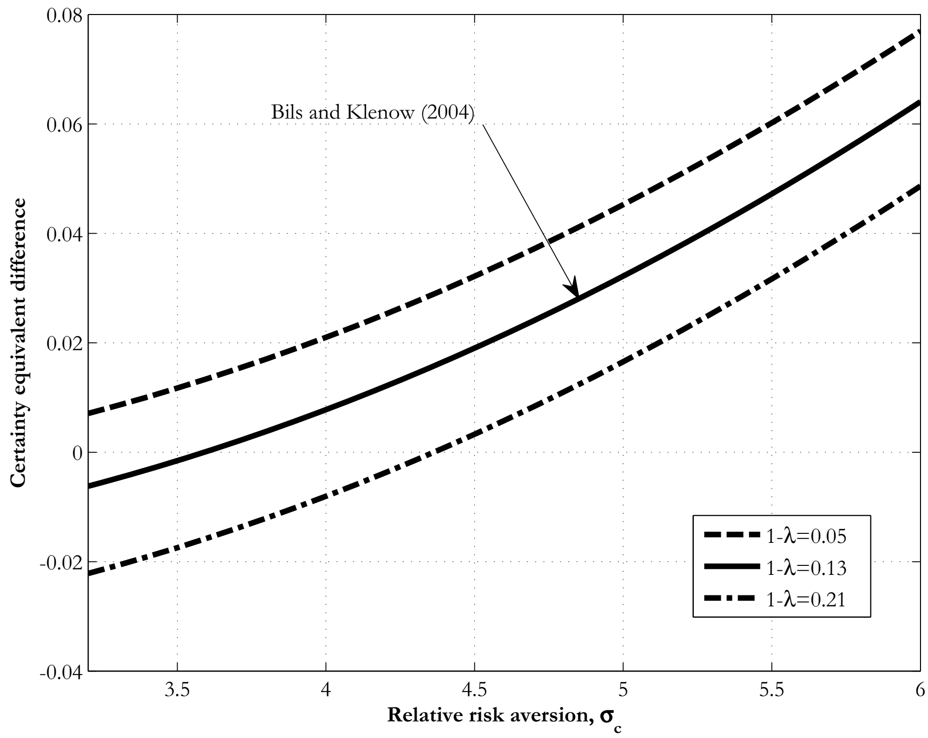

Figure 2.5: The welfare gain of uninformative signals relative to perfectly informative signals as a function risk aversion when households are allowed to reengage in social insurance after one period.

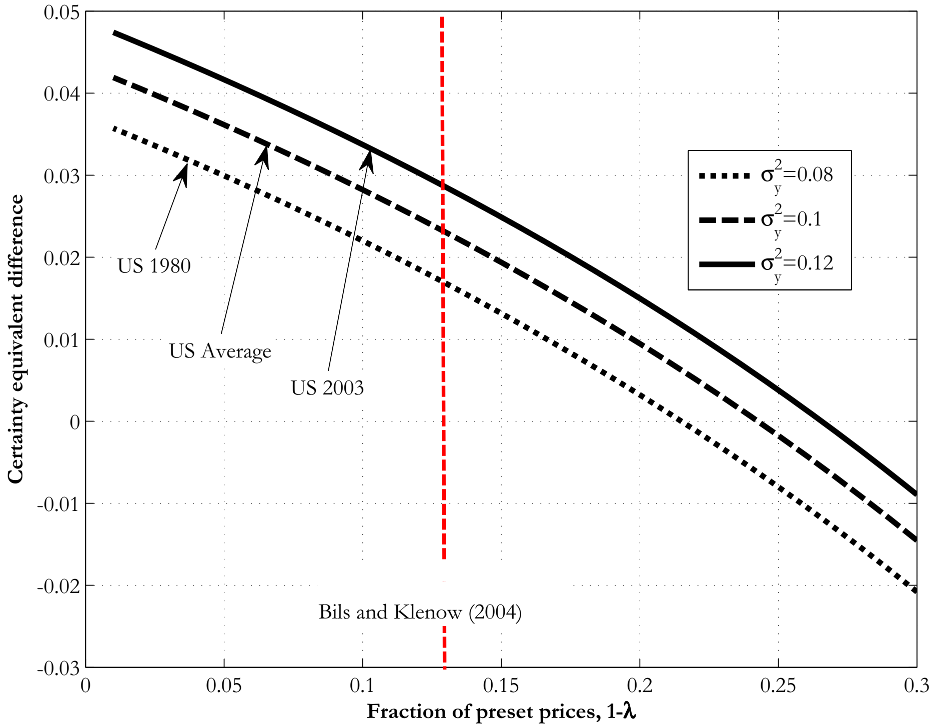

Figure 2.6: The welfare gain of uninformative signals relative to perfectly informative signals as a function the fraction of prices preset when households are allowed to reengage in social insurance after one period.

3 Optimal Interest Rate Stabilization in a Basic Sticky-Price Model

with Matthias Paustian

*T*his chapter studies optimal monetary policy with the nominal interest rate as the single policy instrument. Firms set prices in a staggered way without indexation and real money balances contribute separately to households' utility. The optimal deterministic steady state under commitment is the Friedman rule – even if the importance assigned to the utility of money is small relative to consumption and leisure. We approximate the model around the optimal steady state as the long-run policy target. Optimal monetary policy is characterized by stabilization of the nominal interest rate instead of inflation stabilization as the predominant principle.

3.1 Introduction

What is the primary aim of optimal monetary policy? In the existing literature there are two major views that deliver opposite recommendations for the optimal conduct of monetary policy in the short and in the long run. The first branch goes back to Friedman (1969) and evaluates monetary policy in the long run with fully flexible prices and under perfect competition. In order to equate the private opportunity costs for holding money to the zero social costs to produce it, the nominal interest rate should be zero. The other view considers optimal monetary policy in the short run in the presence of nominal rigidities and imperfect competition (e.g. Benignio and Woodford 2005; Khan et al. 2003; Schmitt-Grohé and Uribe 2004, 2007; Woodford 2003,b). A key feature of this literature is that the authors consider small fluctuations around the (almost) zero inflation steady state, implying that optimal policy nearly completely offsets the distortions due to price dispersion – even in the presence of a monetary friction. The predominant principle is inflation stabilization, while the nominal interest rate should adjust relatively freely to support this principle.

In this paper we revisit the issue of optimal monetary policy in a sticky price model in the presence of a transaction friction. The foremost contribution is to challenge the conventional view that the Friedman rule loses out to the goal of price stability once price stickiness is introduced. We show that the widely used money-in-the-utility-function model (MIU) implies that Friedman's rule is optimal even when large amounts of price stickiness are present. This is in contrast to the key message of papers such as Woodford (2003), Khan et al. (2003), Schmitt-Grohé and Uribe (2004) and others. Second, we find that the primary aim of optimal policy in the short run is to stabilize the nominal interest rate instead of inflation.

Our analysis is set in a dynamic stochastic general equilibrium model with imperfect competition and Calvo's staggered price setting without indexation. A transaction friction is introduced via the textbook money-in-the-utility-function approach following Sidrauski (1967) and more recently Woodford (2003) or Walsh (2003a) with consumption and real money balances entering in a separable way. We abstract from interactions between fiscal and monetary policy by assuming that that the government has access to lump-sum taxes. Since we assume an output subsidy that offsets the steady state distortion created by monopolistic competition, the policy maker faces two distortions: price dispersion due to staggered price setting calls for an optimal inflation of zero, implying costs of money holdings. However, the monetary distortion can only be offset by setting the nominal interest rate to zero.

We choose the long-run target of monetary policy to be the welfare-maximizing deterministic steady state. Remarkably, we find that even for very low values for the weight of money in the utility function relative to consumption and leisure, it is optimal to fully offset the monetary distortion and to allow for a small degree of price dispersion. I.e. the Friedman rule is optimal even in the presence of Calvo-style staggered price setting. This result holds for a wide range of parameter values including low weights for real money balances in the utility function. To understand this finding, note that the welfare cost of price dispersion arising from long-run deflation required by the Friedman rule is small relative to the loss from a positive nominal interest rate. While the welfare loss due to price dispersion hinges primarily on the frequency of price adjustment, the utility loss of a positive interest rate crucially depends on the sensitivity of money demand to the nominal interest rate.

In an MIU framework, the latter increases strongly as interest rates fall. Thereby, the taxation of money holdings via a positive interest rate becomes suboptimal.

We linearize the model around the optimal steady state and derive a quadratic approximation to the utility of the representative household. This welfare based loss function serves as the central bank's objective, and it depends on three arguments: the unconditional variances of inflation, of the output gap, and of the nominal interest rate. While the weight for the variation in the output gap relative to inflation depends exclusively on structural parameters unrelated to policy, the relative weight for interest rate variability also hinges on steady state values that are under control of policy in the long run. Remarkably, the preference to stabilize fluctuations of the nominal interest rate increases as optimal steady state inflation moves towards Friedman's rule of deflation. This increase is primarily driven by the rise in the interest elasticity of money demand. Correspondingly, the importance to account for monetary frictions depends upon the steady state chosen for approximation: The long-run optimal policy is key for optimal policy reactions in the short run. Since we approximate our model around a steady state implied by the Friedman rule, the primary goal of optimal monetary policy is to stabilize variations in the interest rate rather than in inflation. Given the high weight attached to interest rate stabilization, optimal monetary policy requires abstaining from fluctuations in the nominal interest rate. Instead, the nominal interest rate is literally fixed in response to various kinds of disturbances.

We show that choosing a long-run deflation target according to the Friedman rule does not generally undermine the central banks ability to stabilize the welfare relevant fluctuations around that target. On the contrary, the welfare loss arising from fluctuations around the Friedman steady state can be lower than the loss arising from fluctuations around the zero inflation steady state. Overall, we find support for the Friedman rule even in case of a reasonable amount of nominal rigidity due to staggered price setting a la Calvo: The Friedman rule yields higher steady state utility and can also improve welfare effects of fluctuations around the steady state compared to price stability.

We address the issue of the zero bound constraint on the nominal interest rate in the following way. First, we impose that the gross nominal interest rate exceeds unity in the deterministic steady state by a small amount. This assumption does not exclude the possibility of an occasionally binding lower bound constraint in response to shocks. Second, we approximate the probability of hitting the lower bound. We find that it is minor. To be more precise the standard deviation of the nominal interest rate under optimal policy is so small relative to the buffer between the steady state nominal rate and unity that the likelihood of a binding lower bound is low.

Related Literature

We now turn to the related literature. Most closely related to our paper is the work by Woodford (2003, chapters 6-7), and Schmitt-Grohé and Uribe (2007). Woodford also studies optimal monetary policy in a money-in-the-utility-function framework with staggered price setting. In contrast to our analysis, the model is log-linearized around the zero inflation steady state. This approximation point then implies different dynamics for the nominal interest rate. In his analysis, the nominal interest rate reacts rather sharply to shocks while the optimal path of inflation is relatively smooth over the cycle, see Woodford (2003, p. 504). Our contribution is to show that the optimal policy prescriptions differ substantially once one takes into account the interactions between long run and short run optimal policy.

Schmitt-Grohé and Uribe (2007) and Khan et al. (2003) also analyze optimal monetary policy with nominal rigidities and a monetary friction. These papers adopt a transaction technology approach to introducing money into the model. While Khan et al. (2003) use a different time dependent pricing model than we do, the economic environment of Schmitt-Grohé and Uribe (2007) is more similar to our framework. They analyze a medium scale model with staggered price setting a la Calvo and various additional distortions. They find that the central bank should aim at price stability and stabilization of inflation as the main principle. The difference between their key finding and our results is explained as follows. The money-in-the-utility-function approach we employ has different implications for money demand at low interest rates compared to the transactions technology in Schmitt-Grohé and Uribe. The MIU framework implies that the interest elasticity of money demand increases by large amounts as the nominal interest rate approaches the lower bound. Correspondingly, welfare costs of taxing money balances with positive interest rates in the long run and varying the nominal interest rate in the short run increase substantially. This is not the case for their transaction cost technology. Our contribution is to show that both the degree of price dispersion and the sensitivity of money demand with respect to nominal interest rates at low levels are decisive for the conduct of optimal policy.

We are not the first in showing that the Friedman rule can be optimal in economies with sticky prices. Adão et al. (2003) prove that the Friedman rule is optimal in an economy with imperfect competition, a cash in advance constraint, and prices that are set one period in advance. In contrast, our analysis assumes Calvo staggered price setting, which implies that there are costs to deflation due to the dispersion of relative prices. Closer to our work is King and Wolman (1996). They show that setting the nominal interest rate at a minuscule amount above zero maximizes steady state welfare in a model with Calvo pricing and a transaction cost technology. We obtain the same result in an MIU framework. While King and Wolman (1996) focus on a static analysis of optimal policy our main contribution is dynamic: We derive the guiding principles for the optimal conduct of policy in the short run that follow of choosing the Friedman rule as long-run target.

Methodologically, this paper differs from Khan et al. (2003) and Schmitt-Grohé and Uribe (2007) by working with the linear-quadratic framework, rather than with the time invariant Ramsey approach. By showing that the weight on nominal interest stabilization in the loss function depends on the steady state values under control of the central bank, this approach helps to point out intuitively how long run optimal policy and short run stabilization policies are interrelated. In addition, the guiding principle of optimal monetary policy is directly transparent in the size of the relative weights attached to interest rate, inflation, and output gap stabilization.

The remainder of this paper proceeds as follows: in section 2 we set up the model. In section 3 we compute the welfare-maximizing deterministic steady state and derive a quadratic approximation of the utility of the representative household. In section 4 we derive the optimal monetary policy responses in the short run for two policy regimes: the first one has Friedman's Rule, and the other one has zero inflation as its long-run target. The last section concludes.

3.2 The model

We consider an economy that consists of a continuum of infinitely lived households indexed with $j \in [0,1]$. It is assumed that households have identical initial asset endowments and identical preferences. Household j acts as a monopolistic supplier of labor services l_j. Lower (upper) case letters denote real (nominal) variables. At the beginning of period t, households' financial wealth comprises money M_{jt-1}, a portfolio of state contingent claims on other households yielding a (random) payment Z_{jt}, and one period nominally non-state contingent government bonds B_{jt-1} carried over from the previous period. Assuming complete financial markets let $q_{t,t+1}$ denote the period t price of one unit of currency in a particular state of period $t+1$ normalized by the probability of occurrence of that state, conditional on the information available in period t. Then, the price of a random payoff Z_{t+1} in period $t+1$ is given by $E_t[q_{t,t+1}Z_{jt+1}]$. The budget constraint of the representative household reads

$$M_{jt} + B_{jt} + E_t[q_{t,t+1}Z_{jt+1}] + P_t c_{jt} \\ \leq R_{t-1}B_{jt-1} + M_{jt-1} + Z_{jt} + P_t w_{jt} l_{jt} + \int_0^1 D_{jit} di - P_t T_t, \quad (3.1)$$

where c_t denotes a Dixit-Stiglitz aggregate of consumption with elasticity of substitution θ, P_t the aggregate price level, w_{jt} the real wage rate for labor services l_{jt} of type j, T_t a lump-sum tax, R_t the gross nominal interest rate on government bonds, and D_{it} dividends of monopolistically competitive firms. Further, households have to fulfill the no-Ponzi game condition, $\lim_{i\to\infty} E_t q_{t,t+i}(M_{jt+i} + B_{jt+i} + Z_{jt+1+i}) \geq 0$.

The objective of the representative household is

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \{u(c_{jt}, \zeta_t) - v(l_{jt}) + z(M_{jt}/P_t)\}, \quad \beta \in (0,1), \quad (3.2)$$

where β denotes the subjective discount factor and $M_{jt}/P_t = m_{jt}$ end-of-period real money balances. Note that our specification of utility is consistent with recent findings by Andrés, López-Salido, and Vallés (2006) for the Euro area and by Ireland (2004b) for the US. They estimate the role of money for the business cycle and find that preferences are separable between consumption and real money balances.

We assume that households' utility can be affected by a disturbance term ζ_t with mean 1 that can alter the utility of consumption. To avoid additional complexities, we set $u_{c\zeta} = u_c$ at the deterministic steady state. For each value of ζ, the instantaneous utility function is assumed to be non-decreasing in consumption and real balances, decreasing in labor time, strictly concave, twice continuously differentiable, and to fulfill the Inada conditions.

Households are wage-setters supplying differentiated types of labor l_j which are transformed into aggregate labor l_t with $l_t^{(\epsilon_t-1)/\epsilon_t} = \int_0^1 l_{jt}^{(\epsilon_t-1)/\epsilon_t} dj$. We assume that the elasticity of substitution between different types of labor, $\epsilon_t > 1$, varies exogenously over time. The time variation in this markup parameter introduces a so called cost-push shock into the model that gives rise to a stabilization problem for the central bank even in the absence of a transaction friction. Cost minimization implies that the demand for differentiated labor services l_{jt}, is given by $l_{jt} = (w_{jt}/w_t)^{-\epsilon_t} l_t$, where the aggregate real wage rate w_t is given by $w_t^{1-\epsilon_t} = \int_0^1 w_{jt}^{1-\epsilon_t} dj$. Maximizing (3.2) subject to (3.1) and the no-Ponzi game condition for given initial values $M_{t_0-1} > 0$, Z_0, B_{t_0-1}, and $R_{t_0-1} \geq 0$ leads to the following first order conditions for consumption, money, the real wage rate for labor type j, government bonds, and contingent claims:

$$\lambda_{jt} = u_c(c_{jt}, \zeta_t), \quad v_l(l_{jt}) = w_{jt}\lambda_{jt}/\mu_t^w, \tag{3.3}$$

$$\lambda_{jt} - z_m(m_{jt}) = \beta E_t \frac{\lambda_{jt+1}}{\pi_{jt+1}}, \quad q_{t,t+1} = \frac{\beta\lambda_{jt+1}}{\pi_{t+1}\lambda_{jt}}, \quad \lambda_{jt} = \beta R_t E_t \frac{\lambda_{jt+1}}{\pi_{t+1}} \tag{3.4}$$

where λ_{jt} denotes a Lagrange multiplier, π_t the inflation rate $\pi_t = P_t/P_{t-1}$, and $\mu_t^w = \epsilon_t/(\epsilon_t - 1)$ the stochastic wage mark-up with mean $\bar{\mu}^w > 1$. The first order condition for contingent claims holds for each state in period $t+1$, and determines the price of one unit of currency for a particular state at time $t+1$ normalized by the conditional probability of occurrence of that state in units of currency in period t. Absence of arbitrage opportunities between government bonds and contingent claims requires $R_t = 1/E_t q_{t,t+1}$. The optimum is further characterized by the budget constraint (3.1) holding with equality and by the transversality condition $\lim_{i\to\infty} E_t \beta^i \lambda_{jt+i}(M_{jt+i} + B_{jt+i} + Z_{jt+1+i})/P_{jt+i} = 0$.

The final consumption good Y_t is an aggregate of differentiated goods produced by monopolistically competitive firms indexed with $i \in [0,1]$ and defined as $y_t^{\frac{\theta-1}{\theta}} = \int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di$, with $\theta > 1$. Let P_{it} and P_t denote the price of good i set by firm i and the price index for the final good. The demand for each differentiated good is $y_{it}^d = (P_{it}/P_t)^{-\theta} y_t$, with $P_t^{1-\theta} = \int_0^1 P_{it}^{1-\theta} di$. A firm i produces good y_i using a technology that is linear in the labor bundle $l_{it} = [\int_0^1 l_{jit}^{(\epsilon_t-1)/\epsilon_t} dj]^{\epsilon_t/(\epsilon_t-1)}$: $y_{it} = a_t l_{it}$, where $l_t = \int_0^1 l_{it} di$ and a_t is a productivity shock with mean 1. Labor demand satisfies: $mc_{it} = w_t/a_t$, where $mc_{it} = mc_t$ denotes real marginal cost independent of the quantity that is produced by the firm.

We allow for a nominal rigidity in form of staggered price setting as developed by Calvo (1983). Each period firms may reset their prices with the probability $1-\alpha$ independently of the time elapsed since the last price setting. The fraction $\alpha \in [0,1)$ of firms are assumed to keep their previous period's prices, $P_{it} = P_{it-1}$, i.e. indexation is absent. Firms are assumed to maximize their market value, which equals the expected sum of discounted dividends $E_t \sum_{T=t}^{\infty} q_{t,T} D_{iT}$, where $D_{it} \equiv P_{it} y_{it}(1-\tau) - P_t mc_t y_{it}$ and we used the fact that firms also have access to contingent claims. Here, τ denotes an exogenous sales tax introduced to offset the inefficiency of steady state output due to markup pricing as in Rotemberg and Woodford (1999). In each period a measure $1-\alpha$ of randomly selected firms set new prices $\widetilde{P}_{it}$ as the solution to $\max_{\widetilde{P}_{it}} E_t \sum_{T=t}^{\infty} \alpha^{T-t} q_{t,T} (\widetilde{P}_{it} y_{iT}(1-\tau) - P_T mc_T y_{iT})$, s.t. $y_{iT} = (\widetilde{P}_{it})^{-\theta} P_T^{\theta} y_T$. The first order condition for the price of re-optimizing producers is given by

$$\frac{\widetilde{P}_{it}}{P_t} = \frac{\theta}{\theta-1} \frac{F_t}{K_t}, \tag{3.5}$$

where K_t and F_t are defined by the following expressions:

$$F_t = E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} u_c(c_T, \zeta_T^{(1)}) y_T \left(\frac{P_T}{P_t}\right)^{\theta} mc_T \tag{3.6}$$

and

$$K_t = E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} u_c(c_T, \zeta_T^{(1)}) (1-\tau) y_T \left(\frac{P_T}{P_t}\right)^{\theta-1}. \tag{3.7}$$

Aggregate output is given by $y_t = a_t l_t / \Delta_t$, where $\Delta_t = \int_0^1 (P_{it}/P_t)^{-\theta} di \geq 1$ and thus $\Delta_t = (1-\alpha)(\widetilde{P}_t/P_t)^{-\theta} + \alpha \pi_t^{\theta} \Delta_{t-1}$. The dispersion measure Δ_t captures the welfare decreasing effects of staggered price setting. If prices are flexible, $\alpha = 0$, then the first order condition for the optimal price of the differentiated good reads: $mc_t = (1-\tau)\frac{\theta-1}{\theta}$.

The public sector consists of a fiscal and a monetary authority. The central bank as the monetary authority is assumed to control the short-term interest rate R_t as the single instrument. The fiscal authority issues risk-free one period bonds, has to finance exogenous government expenditures $P_t G_t$, receives lump-sum taxes from households, transfers from the monetary authority, and tax-income from an exogenous given constant sales tax τ, such that the consolidated budget constraint reads: $R_{t-1} B_{t-1} + M_{t-1} + P_t G_t = M_t + B_t + P_t T_t + \int_0^1 P_{it} y_{it} \tau di$. The exogenous government expenditures G_t evolve around a mean $\bar{G}$, which is restricted to be a constant fraction of output, $\bar{G} = \bar{y}(1-sc)$. We assume that tax policy guarantees government solvency, i.e., ensures $\lim_{i\to\infty} (M_{t+i} + B_{t+i}) \prod_{v=1}^{i} R_{t+v}^{-1} = 0$. Due to the existence of the lump-sum tax, we consider only the demand effect of government expenditures and focus exclusively on optimal monetary policy.

We collect the exogenous disturbances in the vector $\xi_t = [\zeta_t, a_t, G_t, \mu_t^w]$. It is assumed that the percentage deviation of each of the elements of the vector from their means evolve according to autonomous AR(1)-processes with autocorrelation coefficients $\rho_\zeta, \rho_a, \rho_G, \rho_\mu \in [0,1)$. The innovations are assumed to be i.i.d. The recursive equilibrium is defined as follows:

Definition 3.1. *Given initial values, $M_{t_0-1} > 0$, $P_{t_0-1} > 0$ and $\Delta_{t_0-1} \geq 1$, a monetary policy and a Ricardian fiscal policy $T_t \, \forall \, t \geq t_0$, a sales tax τ, a rational expectations equilibrium (REE) for $R_t \geq 1$, is a set of sequences $\{y_t,\ c_t,\ l_t,\ mc_t,\ w_t,\ \Delta_t,\ P_t,\ \widetilde{P}_t,\ m_t,\ R_t\}_{t=t_0}^{\infty}$ satisfying the firms' first order condition $mc_t = w_t/a_t$, (3.5) with $\widetilde{P}_{it} = \widetilde{P}_t$, and $P_t^{1-\theta} = \alpha P_{t-1}^{1-\theta} + (1-\alpha)\widetilde{P}_t^{1-\theta}$, the households' first order conditions $u_c(y_t - G_t, \zeta_t) w_t = v_l(l_t)\mu_t^w$, $u_c(y_t - G_t, \zeta_t)/P_t = \beta R_t E_t u_c(y_{t+1} - G_{t+1}, \zeta_{t+1})/P_{t+1}$, $z_m(m_t) = u_c(y_t - G_t, \zeta_t)(R_t - 1)/R_t$, the aggregate resource constraint $y_t = a_t l_t/\Delta_t$, where $\Delta_t = (1-\alpha)(\widetilde{P}_t/P_t)^{-\theta} + \alpha(P_t/P_{t-1})^{\theta}\Delta_{t-1}$, clearing of the goods market $c_t + G_t = y_t$ and the transversality condition, for $\{\xi_t\}_{t=t_0}^{\infty}$.*

3.3 The linear-quadratic optimal policy problem

In a first step, we compute the optimal deterministic steady state of the economy as the one that maximizes steady state utility. This steady state is our point of expansion for the log-linear approximation of the model's equilibrium conditions as well as for the derivation of the purely quadratic welfare measure. As we will see, long run and short run optimal policy are closely interrelated. Throughout we assume that the steady state is rendered efficient by an appropriate setting of the tax rate.

3.3.1 The optimal steady state

Our approach to optimal policy in the long run is to maximize steady state utility. Wolman (2001) shows that this criterion gives slightly different prescriptions to optimal policy in the long-run than the time invariant Ramsey concept, but these differences are quantitatively very small. Both approaches differ in our case due to the presence of forward-looking equations and one endogenous state variable, namely price dispersion.

The nonlinear optimization problem for the central bank is to choose steady state values for output, price dispersion, the denominator (K) and the numerator (F), the nominal interest rate and inflation to maximize steady state utility of the representative household

$$\max U = u(y - G, \zeta) - v(\Delta y / a) + z(m(R, y - G, \zeta)), \tag{3.8}$$

subject to the firms' optimal pricing condition, the recursive formulation of the functions K and F, the evolution of the dispersion measures and the euler equation:

$$\rho(\pi)^{\frac{1}{1-\theta}} K = \frac{\theta}{\theta - 1} F \tag{3.9}$$

$$K = u_c(y - G, \zeta)(1 - \tau)y + \beta\alpha K \pi^{\theta - 1} \tag{3.10}$$

$$F = v_l(y\Delta / a)y\mu^w + \alpha\beta F \pi^\theta \tag{3.11}$$

$$\Delta = (1 - \alpha)\rho(\pi)^{\frac{\theta}{\theta-1}} + \alpha\Delta\pi^\theta \tag{3.12}$$

and

$$\pi = \beta R, \tag{3.13}$$

with $\rho(\pi) \equiv (1 - \alpha\pi^{\theta-1})(1 - \alpha)^{-1}$.[1]

To simplify the analysis and to solve for the optimal steady numerically, we assume that households' utility is given by the usual CRRA specification:

$$\frac{c^{1-\sigma_c}}{1 - \sigma_c} - a_2 \frac{l^{1+\omega}}{1 + \omega} + a_1 \frac{m^{1-\sigma_m}}{1 - \sigma_m}, \tag{3.14}$$

σ_c, σ_m positive and ω non-negative. Here, $a_1 \geq 0$ denotes the weight for the utility stemming from real money balances relative to the utility of consumption and a_2 the corresponding relative weight for the disutility of labor.[2] We assume that the zero bound on the nominal interest rate is not binding in expectations. This is equivalent to assuming that inflation in the deterministic steady state is at least $\pi \geq \beta + \epsilon$ for a small parameter $\epsilon > 0$. The reason for this assumption is twofold. Economically, the resulting buffer allows the central bank to adjust its instrument downward in response to a shock (at least by a small amount). Technically, the CRRA preferences do not display a satiation point for real money balances at a finite level. However, by imposing a lower bound on the steady-state nominal interest rate, real money balances are still bounded – even if inflation equals $\beta + \epsilon$ (and $R - 1 = \beta^{-1}\epsilon$). Then all

[1] To simplify the notation, steady-state values in the following are denoted without a time subscript.

[2] The first order conditions and the constraints of the optimization problem of the central bank in the deterministic steady state for the assumed CRRA preferences can be found in appendix 5.2.1.

first and second partial derivatives of utility with respect to c and m exhibit well defined finite limiting values (e.g. $z_{mm} < 0$) as c, m approach their corresponding finite values at the ϵ lower bound, c_ϵ, m_ϵ. In particular, this implies that the interest elasticity of money demand, $\eta_R(R_\epsilon) = z_m(m_\epsilon)[m_\epsilon z_{mm}(m_\epsilon)(R_\epsilon - 1)]^{-1}$ is well-defined and finite since $R_\epsilon - 1 = \beta^{-1}\epsilon$ is a small positive scalar.

In our baseline calibration we set $\theta = 6$ and $\alpha = 0.66$, where the latter can be found for example in Walsh (2005) or Woodford (2003). The parameter a_2 is set such that agents work 1/3 of their available time in the steady state.

We calibrate the money demand block of our model to be in line with the existing literature and U.S. times series data. In particular, we set the annual interest semi-elasticity of money demand, $\partial \log m / \partial R = -[R(R-1)\sigma_m]^{-1}$ equal to - 4.47 at an annual interest rate of R = 1.083. This is in line with Lucas (2000) and Woodford (2003). In calibrating this elasticity we have assumed an average annual inflation rate of 4 per cent together with a real interest rate of 4.3 per cent such that $R = 1.083$. It then follows that $\sigma_m = 2.5$. Note that the semi-elasticity and the elasticity of money demand, $\eta_R(R) \equiv [(R-1)\sigma_m]^{-1} > 0$, increases (in absolute terms) as interest rates decrease. We assume a degree of relative risk aversion $\sigma_c = 2$. This implies an output elasticity of money demand $\sigma_c/(s_c\sigma_m) = 1$. Furthermore, we set the parameter $a_1 = 1/99$ such that at a nominal interest rate of $R = 1.083$ the annual ratio of $M1$ over nominal GDP equals 0.2. This value is consistent with postwar U.S. data and similar to the one used by Schmitt-Grohé and Uribe (2004) and Schmitt-Grohé and Uribe (2007).

Then the following numerical result for the ϵ steady state holds:

Result 3.2. *If* $a_1 \geq 1/3513$ *and the other parameters are given by the baseline calibration, optimal inflation in the deterministic steady state* π *is* $\beta + \epsilon = 0.9901$. *The associated optimal price dispersion* $\bar{\Delta}$ *is* 1.0014, *while the optimal nominal interest rate* $\bar{R}$ *is* $1.0001 > 1$.

Under the baseline calibration, we find that the optimal steady-state value for inflation is the lower bound, $\pi = \beta + \epsilon$, i.e. it involves deflation. Correspondingly, the nominal interest rate is almost zero.

Since a_1 is an unobserved preference parameter, it is difficult to assess whether the critical value $a_1 = 1/3513$ implies a large or small role for money in the utility function. However, the annual steady state ratio of $M1$ over nominal GDP implied by this critical value is 0.048. Hence, even if the importance of money in transactions - as measured by this ratio - falls by 76% from its baseline value of 0.2, the Friedman rule would still be optimal. Therefore, the Friedman rule is optimal in our model even when money provides a very small flow of utility.

Why does the Friedman rule turn out to be optimal even when the importance of real money balances in the utility function is very low? Optimal monetary policy seeks to minimize two distortions created by price dispersion and the transaction friction, since the monopolistic distortion is eliminated in the steady state by an output subsidy.[3] Price dispersion calls for an inflation rate of zero, while the monetary friction requires deflation. Correspondingly, we expect our optimal gross inflation rate to be found between β and unity. While studies such as Kiley (2002) and Ascari (2004) have shown that relatively small amounts of trend inflation are associated with relatively large welfare costs under Calvo pricing, this is not the case for long run deflation. Figure 4 in the appendix shows that the price dispersion arising from long run deflation is relatively small. The second reason for the optimality of Friedman's rule is an adaption of a general principle of optimal taxation in public finance. Since the interest rate acts like a tax on money holdings, it should be low due to the fact that money demand is elastic with respect to interest under price stability. While the choice for ϵ is arbitrary, our results are not very sensitive to the magnitude of ϵ (see Figure 1). The graph plots optimal annual inflation against the degree of price dispersion α. Remarkably, our threshold levels for the optimality of Friedman's rule differ substantially from the results obtained by Schmitt-Grohé and Uribe (2007, Figure 1). While the Friedman rule in our model is optimal until the degree of price dispersion reaches 0.81, Schmitt-Grohé and Uribe find a considerably lower breaking point of approximately 0.46 (see the vertical line in Figure 1), since the welfare costs of positive interest rates are lower in their transaction costs specification. To be more precise the MIU framework, unlike theirs, implies that the interest elasticity of money demand increases by large amounts as the nominal interest rate approaches the lower bound.[4]

Which parameters influence the lower bound on a_1, i.e. the minimum weight for money in the utility function that renders the Friedman rule optimal? Put differ-

[3]The output subsidy of $\tau = 1 - (1-\alpha\beta\pi^{\theta-1})\mu^{w}\theta\rho(\pi)^{1/(\theta-1)}[(1-\alpha\beta\pi^{\theta})(\theta-1)]^{-1} < 0$ depends on steady state deflation. However, this feature does not favor the Friedman Rule in the steady state. If we were to apply the subsidy under zero-inflation, $\tau = 1 - \mu^{w}\theta/(\theta-1)$, the Friedman Rule would be optimal for even smaller relative weights of money in the utility function. The reason is as follows. Note that steady state output is lower when the subsidy does not depend on trend deflation. Note further that the utility loss that households suffer due to a positive steady state price dispersion is weighted with the steady state output.

[4]While not uncontroversial this property is not special to our MIU formulation. It can be obtained in non-separable MIU specifications as well as in MIU models with a satiation point for real money balances and in transactions cost models.

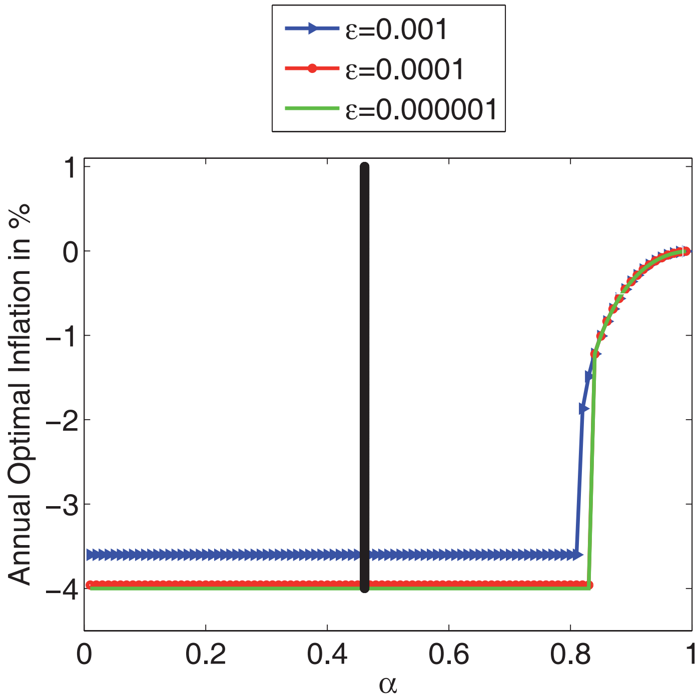

Figure 3.1: Optimal annual inflation and Calvo parameter α. The vertical line denotes the critical value in Schmitt-Grohé and Uribe (2007) for which the Friedman rule ceases to be optimal.

ently, which structural features work in favor for the Friedman rule and when does price dispersion become the main focus of monetary policy? To gain intuition for this question, we compare the outcomes of the Friedman rule and a zero inflation policy and derive an analytical expression of the threshold for which the former dominates the latter policy.

Proposition 3.3. *Assume that preferences are of the separable CRRA type and logarithmic,* $\sigma_m = \sigma_c = 1$*, and* $a_2 = 1$*. Then the Friedman Rule steady state,* $\pi_{FR} = \beta + \epsilon$*, yields higher utility than the zero inflation steady state,* $\pi_{ZERO} = 1$*, if and only if*

$$a_1 > \underline{a_1} \equiv \frac{\frac{\Delta_{FR}-1}{(1+\omega)sc} + \frac{\omega}{1+\omega}\ln[\Delta_{FR}]}{\ln[R_{FR}\eta_{R,FR}(R_{ZERO}\eta_{R,ZERO})^{-1}] - \omega/(1+\omega)\ln[\Delta_{FR}]}$$

with Δ_{FR} *as the price dispersion associated with* $\pi = \beta + \epsilon$ *and*
$R_{FR}\eta_{R,FR}(R_{ZERO}\eta_{R,ZERO})^{-1} = (1-\beta)(1+\beta^{-1}\epsilon)/\beta^{-1}\epsilon.$

Proof see appendix 5.2.2.

$R_{ZERO} = \beta^{-1}$ and $R_{FR} = 1 + \beta^{-1}\epsilon$ denote the gross nominal interest rate under zero inflation and Friedman's rule. Evidently, the Friedman rule performs better than a zero inflation regime, when the degree of price dispersion associated with the Friedman rule, Δ_{FR} is small. But at least equally important is the sensitivity of money demand with respect to interest rates under Friedman's rule, $\eta_{R,FR}$, compared to the corresponding elasticity if zero inflation applies, $\eta_{R,ZERO}$. If these elasticities differ substantially, the amount and utility of real money balances in both regimes differ, too. As will become clear below, this elasticity heavily influences the possible welfare losses due to positive interest rates. Furthermore, a large fraction of private consumption, sc, favors the Friedman rule. The intuition is as follows. Consider a value for a_1 such that the Friedman rule delivers the same steady state welfare as the zero inflation policy. If the fraction of government expenditures decreases, people have to work less since less output has to be produced. Due to price dispersion, people work more under the Friedman Rule, such that their marginal disutility of labor is always higher than under the zero inflation regime. Correspondingly, a one percent decrease in labor in both regimes leads to relatively larger utility gains in the Friedman Rule regime.

In the following subsection we consider optimal monetary policy in the short run, assuming the baseline calibration, such that $\beta + \epsilon$ is the optimal inflation rate.

3.3.2 Approximating the model around the optimal steady state

The model is log-linearized around the optimal deterministic steady state $\pi = \beta + \epsilon < 1$, i.e. under trend deflation and closely follows the approximation around trend inflation as in Ascari (2004). The rational expectations equilibrium for the log-linear-approximate model is then a set of sequences $\{\widehat{y}_t, \widehat{\pi}_t, \widehat{m}_t, \widehat{R}_t, \widehat{F}_t\}_{t=t_0}^{\infty}$ consistent with the following set of equilibrium conditions: the Euler equation, the money demand function, the aggregate supply curve, and the numerator in (3.5)

$$\sigma(E_t\widehat{y}_{t+1} - \widehat{y}_t + g_t - g_{t+1}) = \widehat{R}_t - E_t\widehat{\pi}_{t+1}, \tag{3.15}$$

$$\widehat{m}_t = \frac{\sigma}{\sigma_m}(\widehat{y}_t - g_t) - \eta_{R,FR}\widehat{R}_t, \tag{3.16}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \kappa^*(\omega+\sigma)(\widehat{y}_t - \widehat{y}_t^z) + \frac{\kappa^*(\bar{\pi}-1)}{1-\alpha\beta\pi^\theta}[(\sigma-1)\widehat{y}_t + \widehat{F}_t] \tag{3.17}$$

$$\widehat{F}_t = (1-\alpha\beta\pi^\theta)[(1+\omega)\widehat{y}_t + \widehat{\mu}_t^w - (1+\omega)\widehat{a}_t)] + \alpha\beta\pi^\theta E_t(\theta\widehat{\pi}_{t+1} + \widehat{F}_{t+1}), \tag{3.18}$$

where $\eta_{R,FR} = [\sigma_m(R_{FR}-1)]^{-1}$, $sc = c/y$, $\sigma_c = -u_{cc}c/u_c c > 0$, $\sigma = \sigma_c sc^{-1}$, $\omega = v_{ll}l/v_l > 0$, $g_t = (G_t - G)/y + \sigma^{-1}\widehat{\zeta}_t$, $\kappa^* = (1-\alpha\pi^{\theta-1})(1-\beta\alpha\pi^\theta)/(\alpha\pi^\theta)$, disturbances are collected in

$$\widehat{y}_t^z = ((1+\omega)\widehat{a}_t + \sigma g_t - \widehat{\mu}_t^w)/(\omega+\sigma),$$

$\sigma_m = -z_{mm}(\bar{m})\bar{m}/z_m(\bar{m}) > 0$, the transversality condition, for a monetary policy, a sequence $\{\widehat{\xi}_t\}_{t=t_0}^{\infty}$, and given initial values M_{t_0-1} and P_{t_0-1}. [5] Further $\widehat{z}_t$ denotes the percent deviation of a generic variable z_t from its steady-state value z. In addition we assume that the bounds on the fluctuations of the shock vector $\|\log \xi_t\|$ are sufficiently tight, such that ξ_t remains in the neighborhood of its steady-state value.

3.3.3 A quadratic policy objective

In this section we derive a purely quadratic welfare measure for the utility of the average household as the relevant objective for optimal monetary policy in the short run.

We assume that the welfare-relevant objective is the expected and discounted average utility level of all households, which is given by

$$U_{t_o} \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{u(c_t, \zeta_t) - \int_0^1 v(l_{jt})dj + z(M_t/P_t)\}. \tag{3.19}$$

Our aim is to derive a quadratic loss function that yields an accurate second order approximation of the average utility of all households. We seek to evaluate the approximated level of utility by using the log-linearized conditions (3.15)-(3.18) describing the competitive equilibrium – that is, we set up the familiar linear-quadratic optimal policy problem. A correct welfare ranking of alternative policies requires a second-order approximation of utility that involves no linear terms – at least in expectations, see Woodford (2003, ch. 6).

The existence of a non-zero linear term in the utility approximation crucially relies on the distortions of the steady state output relative to the efficient output level as consequences of price and wage-setting power, distortionary taxation and trend deflation that are represented in ϕ:

$$1-\phi = \rho(\pi)^{\frac{1}{1-\theta}}(1-\tau)\frac{\theta-1}{\mu^w\theta}\frac{1-\alpha\beta\pi^{\theta}}{1-\alpha\beta\pi^{\theta-1}} = \frac{v_l}{u_c}. \tag{3.20}$$

[5] The derivation of the aggregate supply curve can be found in appendix 5.2.3.

If this inefficiency gap is zero or only of first order in ϕ, the linear term in the second order approximation vanishes. Following Rotemberg and Woodford (1999) we assume that the sales tax plays a role of an output subsidy that offsets exactly the steady state output distortion. Since we assume separability between consumption and real money balances, this implies that real balance effects do not contribute to this inefficiency measure.

As Carlstrom and Fuerst (2004) point out, the inclusion of money demand fundamentally changes optimal monetary policy responses even if one assumes – as we do – that real balances do not affect the dynamic evolution of inflation and output in the competitive equilibrium. The reason is that variations in the nominal interest rate contribute to the relevant distortions the policy maker seeks to stabilize. As we will show below, the relative weight of variations in the interest rate that enters the welfare measure is substantially increased if we approximate around the optimal steady state. In the following Proposition we derive a quadratic Taylor-series approximation to (3.19).

Proposition 3.4. *If the fluctuations in y_t around y, R_t around R, ξ_t around ξ, π_t around π are small enough, π and Δ are close enough to 1, and if the steady state distortions ϕ vanish due to the existence of an appropriate subsidy τ, the utility of the average household can be approximated by:*

$$U_{t_0} = -\Omega E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\lambda_x (\widehat{y}_t - \widehat{y}_t^*)^2 + \widehat{\pi}_t^2 + \lambda_R \widehat{R}_t^2] + t.i.s.p. + \mathcal{O}(\|\widehat{\xi}_t, \varsigma\|^3), \quad (3.21)$$

where t.i.s.p. indicate terms independent of stabilization policy, $\kappa = (1-\alpha)(1-\alpha\beta)(\omega + \sigma)/\alpha$, $\Omega = \frac{u_c y \theta(\omega+\sigma)}{2\kappa}$,

$$\lambda_x = \frac{\kappa}{\theta}, \quad (3.22)$$

$$\lambda_R = \frac{\eta_{R,FR}\lambda_x}{v(\omega+\sigma)}, \quad (3.23)$$

and

$$\widehat{y}_t^* = \frac{\sigma g_t + (1+\omega)\widehat{a}_t}{\omega+\sigma}, \quad (3.24)$$

where $v = y/m > 0$ and $\eta_{R,FR}$ is the interest elasticity of money demand at the Friedman rule steady state.

Proof see appendix 5.2.4.

Under the conditions given in Proposition 3.4, the relative weights of inflation, output gap and the nominal interest rates correspond to the results in Woodford (2003). Our analysis differs from Woodford (2003), because the steady state values relate to the lower bound and no longer to price stability as in his analysis.

Remarkably, only the weight to stabilize fluctuations in the nominal interest rate depends on steady state values, v and $\eta_{R,FR}$. Since we approximate our model around the deterministic steady state consistent with the Friedman Rule, the value for the former is small and the value for the latter is large, implying a high preference to stabilize variations in the opportunity costs to hold money.

To set up the optimal policy problem, we need to rewrite the relevant constraints, i.e. the Euler-equation, the law of motion for $\widehat{F}_t$ and the aggregate supply curve in terms of the welfare-relevant output gap, $x_t = \widehat{y}_t - \widehat{y}_t^*$:

$$\widehat{R}_t = E_t\widehat{\pi}_{t+1} + \sigma(E_t x_{t+1} - x_t) + n_t, \tag{3.25}$$

$$\widehat{F}_t = (1 - \alpha\beta\bar{\pi}^{\theta})(1+\omega)x_t + u_t + \alpha\beta\bar{\pi}^{\theta}E_t(\theta\widehat{\pi}_{t+1} + \widehat{F}_{t+1}) \tag{3.26}$$

and

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \eta_4 x_t + \frac{\kappa^*(\bar{\pi}-1)}{1-\alpha\beta\bar{\pi}^{\theta}}\widehat{F}_t + s_t. \tag{3.27}$$

Here, n_t, u_t, s_t denote linear combinations of the elements of $\widehat{\xi}_t$ and η_4 is a constant, which are defined in appendix 5.2.5. Note, that the money demand condition does not enter the set of relevant constraints of the policy problem. Nevertheless it influences the optimal decision via the quadratic loss function, in which it plays an important role in determining the relative weight of interest rate variations.

3.4 Optimal monetary policy in the short run

Our approach to optimal policy in the short run is the timeless perspective. At $t = t_0$ the central bank respects prior commitments made in the infinite past, see Woodford (2003). Hence, the associated optimality conditions will be time invariant which marks the difference to a standard commitment approach. In particular, the optimality conditions in the initial period do not differ from those in later periods. We showed that the optimal policy in the long run is to follow the Friedman rule. In this section we consider the implications for optimal policy in the short run, if deflation – instead of zero inflation – is chosen as the optimal long-run target. In particular, we consider the optimal reaction to various kinds of disturbances and evaluate the resulting stabilization loss of both regimes.

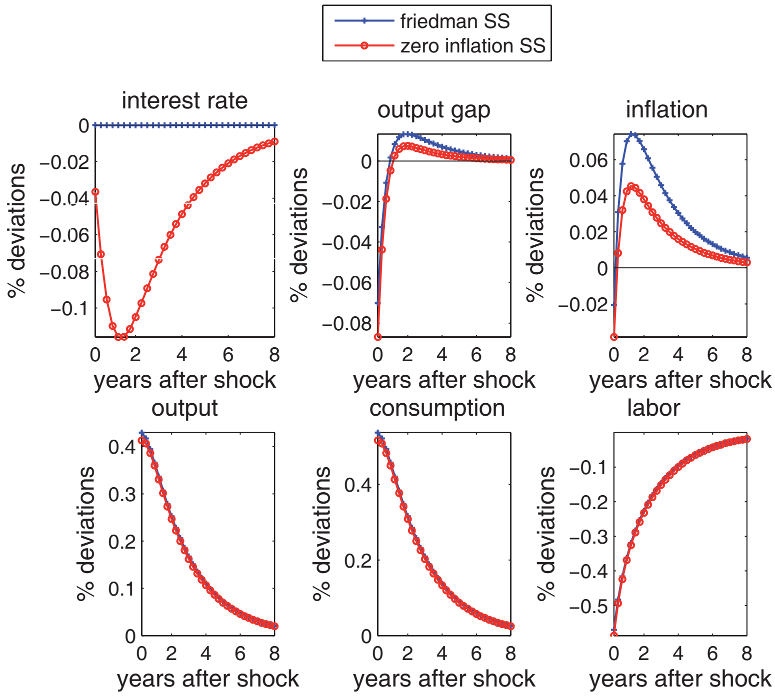

Figure 3.2: Optimal responses to a technology shock

3.4.1 Optimal response to shocks

Our impulse responses analysis distinguishes two cases. In the first case, our set of equilibrium conditions is log-linearized around the optimal steady state in which the inflation rate is equal to $\beta + \epsilon$. In the second case, we follow the conventional procedure and approximate around a steady state of zero inflation. The choice of a point of expansion for the log-linearization affects both the loss function and equilibrium conditions. Log-linearizing around the Friedman rule increases the relative weight on the stabilization of the nominal interest rate and affects the coefficients in the Phillips curve.

When we log-linearize around the optimal steady state corresponding to the Friedman rule, we find that the central bank essentially keeps the nominal interest rate fixed in response to any of the shocks present in our model. Consider first the optimal response to a technology shock displayed in Figure 2. A first-order Taylor expansion around zero inflation suggests that the central bank should lower the annualized nominal rate by roughly 12 basis points and then gradually return to the steady state. However, linearization around the Friedman rule implies that the nominal rate is literally fixed. In line with this finding, the approximation around the Friedman rule implies more volatile response of inflation and the output gap

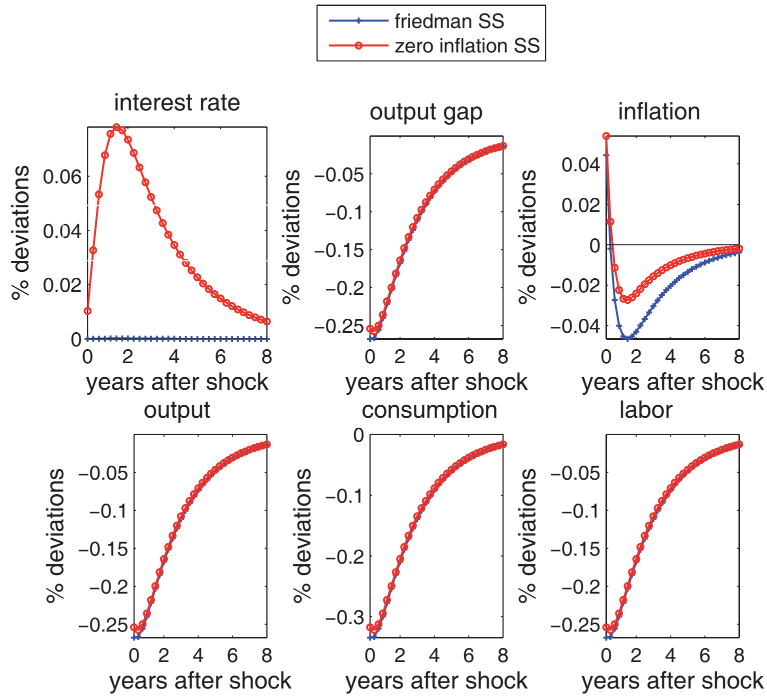

Figure 3.3: Optimal responses to a wage markup shock

than what is suggested by linearization around the zero inflation steady state. A stronger stabilization of the nominal interest rate necessarily implies that the other arguments in the loss function can only be stabilized less. Impulse responses to the other shocks deliver a similar message: Linearization around the Friedman steady state implies that the nominal interest rate is literally fixed. To understand this, note that the interest elasticity of money demand, $[\sigma_m(R-1)]^{-1}$ becomes very large as R approaches its lower bound. For our baseline calibration this elasticity is roughly -4000 at $R = 1 + \beta^{-1}\epsilon$. Despite the fact that the marginal utility of real balances is close to zero, this large elasticity explains why the central bank wishes to hold the nominal rate constant under the Friedman rule.

When deriving the quadratic policy objective we need to assume that price dispersion in the steady state was small. Does this assumption hold in our model? Note from Figure 4 in appendix 3.5 that the dispersion measure is lower for deflation than for inflation. Hence, the condition is more likely to be fulfilled when the model is approximated around a deflationary steady state.[6] To further reassure the reader, we compare the impulse response functions from the linear-quadratic approach to those obtained from linearizing the first order conditions of the non-linear time invariant Ramsey problem (5.39)-(5.44), as well as the constraints (5.45)-(5.49) and log-

[6]This depends crucially on the absence of strategic complementarities in price setting, see Levin, López-Salido, and Yun (2006).

linearize them around the optimal steady state (see appendix 5.2.6).[7] The results of this experiment are displayed in Figure 5 in appendix 3.5. The impulse responses are remarkably similar indicating the accuracy of our linear quadratic approach.[8]

3.4.2 Welfare analysis

In this subsection we compare the welfare implications of the two policy regimes – the long run deflation target according to the Friedman rule vs. zero inflation as the long-run target. Using (3.21) a second-order accurate approximation to the utility of the average household is given by:

$$U_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} U_t \approx \frac{1}{1-\beta}\bar{U} - \Omega E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \lambda_x (\widehat{y}_t - \widehat{y}_t^*)^2 + \widehat{\pi}_t^2 + \lambda_R \widehat{R}_t^2. \quad (3.28)$$

The first part, the discounted steady state utility, is shown to be higher if the Friedman rule is optimal. The second part, the stabilization loss, that relates to the optimal policy reaction in the short run, is not necessarily lower under the Friedman rule regime than under zero inflation. Which of those two parts dominates depends on the calibration of the model, e.g. increasing the variances of the innovations amplifies the welfare loss due to short run fluctuations. In line with the spirit of the timeless perspective, we do not compute welfare conditional on a particular initial state vector at time t_0. Our short run stabilization loss is given by the discounted and weighted sum of unconditional variances:

$$SL = -\frac{1}{1-\beta}\Omega\{var(\widehat{\pi}) + \lambda_x var(x) + \lambda_R var(\widehat{R})\} = -\frac{1}{1-\beta}\Omega L, \quad (3.29)$$

Here L is proportional to the unconditional expectation of period utility.

In Table 2 below we list the relative loss differences under the two policy regimes for a range of relative weights for the utility of real money balances given our baseline calibration for other parameters. For this purpose we calibrate the stochastic shock processes to match the standard deviations of real private consumption and

[7] We follow a procedure proposed by Khan et al. (2003), which is more recently applied in Schmitt-Grohé and Uribe (2007).

[8] To induce the system of first-order conditions of the Ramsey planner to have the same steady state as the one chosen as our expansion point of the linear-quadratic problem, we have to add a constant to the first-order condition of the Ramsey planner for the nominal interest rate that is non-zero. This constant picks up the steady-state slack that arises because the Friedman rule steady state constitutes a corner solution. The constant plays no further role for the dynamics.

government spending of U.S. data during the post-Volcker period.[9] All exogenous processes are assumed to be autocorrelated with coefficient 0.9. We have chosen a standard deviation of the innovations to the taste shock of 0.0001, for the markup shock 0.00015, for the government spending shock 0.0075 and for the technology shock 0.0096.

The results in Table 2 show that the larger the preference parameter a_1 the larger is the weight for interest rate stabilization in the loss function, λ_R. Here, λ_R^{ZERO} denotes the weight when the model is approximated around the zero inflation and λ_R^{FR} denotes the weight for the approximation around the Friedman rule steady state.

The fourth column displays the difference in stabilization loss or business-cycle costs under both regimes. To be more precise, it depicts how much steady state consumption agents are willing to give up permanently to compensate for short run fluctuations in the regime with zero inflation as long-run target, bcc^{ZERO}, relative to the Friedman rule regime, bcc^{FR}. Evidently, this difference is small, e.g. 0.0005% under the baseline calibration with $a_1 = 1/99$. The resulting stabilization loss, when approximating around the Friedman rule steady state is superior to the stabilization loss around zero inflation if a_1 is large enough.

The (technical) intuition for this is a trade off effect between predictability and possible welfare losses in the neighborhood of the steady state of each regime. If the Friedman rule is the expansion point, then the reduced form involves 4 jump variables, $\widehat{R}_t$, x_t, $\widehat{\pi}_t$ and $\widehat{F}_t$, as well as 3 endogenous state variables, the multipliers on the relevant constraints, (3.25)-(3.27). If zero inflation is chosen as the approximation point, the reduced form does not involve $\widehat{F}_t$ and exhibits only the two multipliers associated with the aggregate supply curve and the euler equation as endogenous state variables. On the one hand, the fundamental solution in the Friedman regime is characterized by an additional history dependent variable. This tends to increase prediction power by reducing the forecast error variances of inflation, output gap and the nominal interest rate.[10] On the other hand, however, possible welfare losses in the neighborhood of the zero inflation steady state are lower, steady state utility is "flatter" around $\pi = 1$ (see Figure 6 in appendix 3.5). If the relative weight of

[9]The quarterly data is logged and detrended via the Hodrick-Prescott filter with a smoothing parameter of 10,000. The obtained standard deviation of private consumption is 0.0123, for government expenditures we obtain 0.0172.

[10]E.g. Woodford (2003b) or Walsh (2003b) find that history-dependence can be beneficial for social welfare in forward-looking models. Note however, that in our case the models are not structurally nested, since in the Friedman regime more jump variables must be pinned down.

real money balances decreases, the additional state variable loses prediction power, while possible welfare losses around the zero inflation steady state decrease.

While there is a cut-off value in terms of stabilization loss, overall utility composed of steady state utility minus stabilization loss, is higher under the Friedman rule than under zero inflation. The third but last column of Table 2 depicts this overall difference in utility (denoted by dU_{CE}) expressed in steady state consumption equivalents of the Friedman rule steady state. Under the baseline calibration ($a_1 = 1/99$) agents are willing to give up permanently 0.64% of their consumption in the Friedman rule steady state until they are indifferent between the Friedman rule and the zero inflation regime.

We address the issue of the lower bound approximately in a way proposed originally by Rotemberg and Woodford (1997) and more recently by Schmitt-Grohé and Uribe (2007). First, we compute the optimal steady state under the assumption that the steady state nominal interest rate is at least slightly positive, $R - 1 = \beta^{-1}\epsilon > 0$ in the Friedman rule regime, and $R = \beta^{-1} > 0$ under zero inflation. However, this does not exclude an occasionally binding zero bound. The entries σ_{FR} and σ_{ZERO} shed light on how likely it is that the lower bound on the nominal interest rate binds if the economy fluctuates around the Friedman rule ϵ steady state or around price stability. We calculate the standard deviation of the nominal interest rate under the optimal policy implied by both policy regimes. The term σ_{FR} then expresses the size of the interval from $R = 1.0001$ to the lower bound $R = 1$ in terms of this standard deviation. The entry σ_{ZERO} does the same, but now the approximation is computed around a zero inflation steady state. Hence, larger values for σ_{FR} or for σ_{ZERO} imply that the lower bound is less likely to be binding. Note that our results imply a low probability that the nominal interest rate hits the lower bound. Even for a small relative weight of real money balances, $a_1 = 1/1000$, the resulting standard deviation for the nominal interest rate is small relative to ϵ. A symmetric confidence interval around $R = 1.0001$ of up to 66 standard deviations could be constructed until the lower bound is included. If we decrease ϵ, i.e. if the assumed lower bound moves closer to zero, the corresponding number of standard deviations increases to 418 (see Table 3 in appendix 3.5). This implies that the effect to stabilize the nominal interest (higher relative weight λ_R^{FR}) dominates the effect of the smaller distance to the zero bound. Correspondingly, if zero inflation is chosen as the expansion point, the probability to hit the lower bound is even higher (see the last column). We stress that our attempt to approximating the probability of lower bound violations ignores certain feedback channels.[11] Nevertheless, computing the variance of the nominal interest rate is one way to gauge the severity of the lower bound constraint in linear models.

3.5 Conclusion

We study optimal monetary policy in an economy without capital, where firms set prices in a staggered way without indexation and real money balances are assumed to provide utility. Accounting for a sizeable degree of nominal rigidity, the optimal deterministic steady state that maximizes steady state utility is to follow the Friedman rule, even if the importance assigned to the utility of money is small relative to consumption and leisure.

We approximate the model around this optimal steady state as the long-run policy target and derive a second order approximation to households' utility. Optimal interest rate policy is shown to abstain from reacting sharply to changes in the state of the economy. Instead of stabilizing inflation, the primary goal of the central bank is to stabilize fluctuations in the nominal interest rate.

We stress that our model is not about direct and quantitative advice on optimal monetary policy. It is too stylized for this purpose. The foremost contribution of this paper is to challenge the conventional view that the Friedman rule loses out to the goal of price stability once price stickiness is introduced. We show that a widely used money-in-the-utility-function model implies that the Friedman rule is optimal even when large amounts of price stickiness are present. When the economy fluctuates around the Friedman rule steady state, central bankers should keep the nominal interest stable over the business cycle. This result is explained by the large interest elasticity of money demand that is obtained in our MIU model when the nominal rate is close to zero. There is little empirical evidence on the behavior of money demand in the major industrialized countries for very low interest rates. This is unfortunate as the interest elasticity at low interest rates is a key difference between our MIU framework and the transactions technology employed in other papers that come to different policy prescriptions. Therefore, future research on optimal monetary policy in sticky price models benefits from a better understanding of money

[11] Recent work by Adam and Billi (2006) or Eggertsson and Woodford (2003) explicitly accounts for the non-linear lower bound constraint and shows how the possibility of a binding constraint affects agents' decisions. They find, that the lower bound constraint may amplify the adverse effects of shocks and trigger a stronger response of monetary policy.

demand. Recent work by Ireland (2007) contributes to this issue and points towards a change in U.S. money demand at low rates in the post 1980 period.

A.3 Additional figures

σ_c	σ_m	ω	β	a_1	a_2	$sc = \bar{c}/\bar{y}$	$\bar{\mu}^w$	θ	α	ϵ
2	2.5	0.5	0.99	1/99	25	0.8	7/6	6	0.66	0.0001

Table 3.1: Baseline calibration

a_1	λ_R^{ZERO}	λ_R^{FR}	$bcc^{ZERO} - bcc^{FR}$	dU_{CE}	σ_{FR}	σ_{ZERO}
1/20	2.3426	1472	0.0017%	1.3617%	317	41
1/50	1.6238	1020	0.0011%	0.8959%	220	31
1/99	1.2355	777	0.0005%	0.6423%	167	25
1/150	1.0463	658	0.0002%	0.5182%	142	23
1/189	0.9539	600	0.00%	0.4574%	129	21
1/250	0.8530	536	−0.0002%	0.3909%	116	20
1/500	0.6464	406	−0.0008%	0.2544%	88	17
1/1000	0.4899	308	−0.0014%	0.1506%	66	15

Table 3.2: Welfare Analysis: $\epsilon = 0.0001$

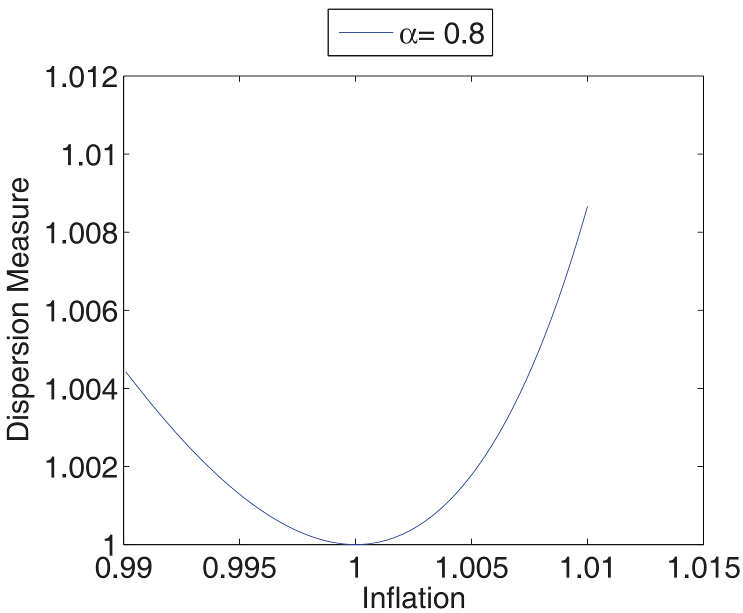

Figure 3.4: Steady state price dispersion as a function of inflation.

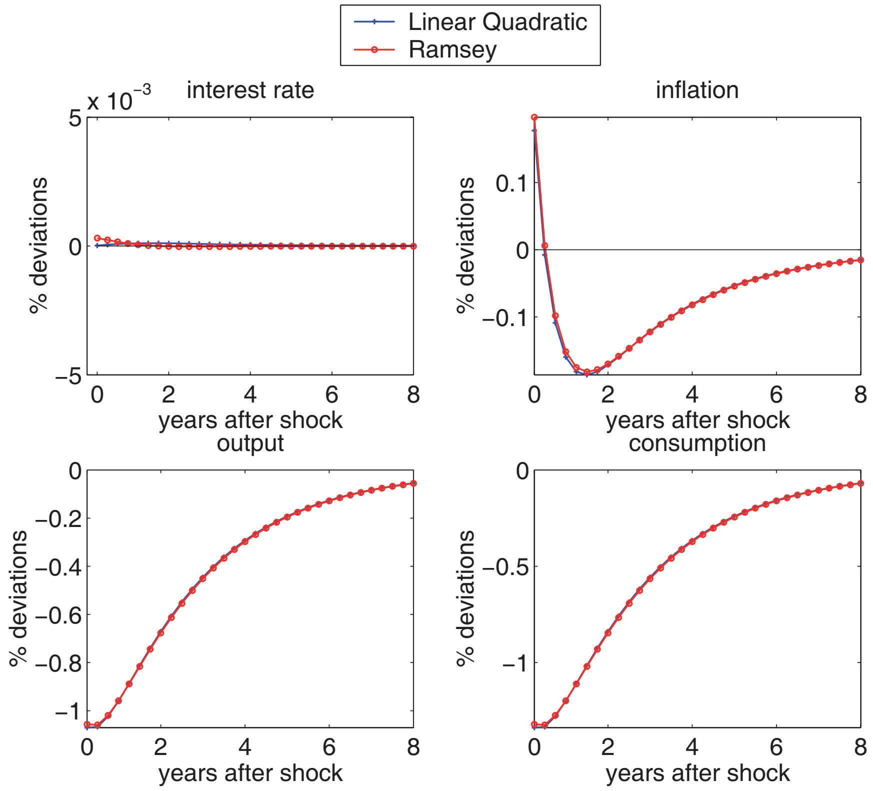

Figure 3.5: Impulse responses to a wage markup shock under optimal policy computed by the linear quadratic approximation and by the time invariant Ramsey approach.

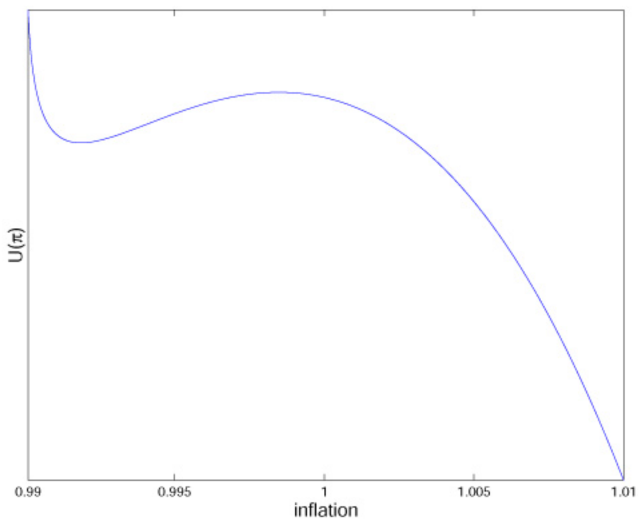

Figure 3.6: Welfare and inflation in the steady state for $\sigma_c = \sigma_m = 1$ and $a_1 = 1/2500$

a_1	λ_R^{ZERO}	λ_R^{FR}	$bcc^{ZERO} - bcc^{FR}$	dU_{CE}	σ(FR)	$\sigma(ZERO)$
1/20	2.3426	928920	0.0017%	1.4543%	1999	41
1/50	1.6238	643870	0.0011%	0.9598%	1386	31
1/99	1.2355	488930	0.0005%	0.6904%	1054	25
1/150	1.0463	414910	0.0002%	0.5585	893	23
1/189	0.9539	378270	0.00%	0.4940%	814	21
1/250	0.8530	338230	-0.0002%	0.4233%	728	20
1/500	0.6464	256330	-0.0009%	0.2783%	552	17
1/1000	0.4899	19460	-0.0015%	0.1680%	418	15

Table 3.3: Welfare Analysis: $\epsilon = 0.000001$

4 Real Balance Effects, Timing and Equilibrium Determination

*T*his paper examines whether the existence and the timing of real balance effects contribute to the determination of the absolute price level, as suggested by Patinkin (1949, 1965). As the main novel result, I show that there exists a unique price level sequence that is consistent with an equilibrium under interest rate policy, if beginning-of-period money yields transaction services. Predetermined real money balances can then serve as a state variable, implying that interest rate setting must be passive – a violation of the Taylor-principle – for unique, stable, and non-oscillatory equilibrium sequences.

4.1 Introduction

The conduct of monetary and fiscal policy is known to affect the determination of the price level and, under non-neutrality, the real equilibrium allocation. While previous contributions to this line of research have primarily considered monetary policy regimes that are characterized by constant money growth (see Matsuyama 1990, 1991; Obstfeld and Rogoff 1983), recent studies mainly focus on policy regimes summarized by interest rate feedback rules, such as Taylor (1993), Benhabib et al. (2001a); Benhabib, Schmitt-Grohé, and Uribe (2001b, 2003), Woodford (1994, 2003) or Carlstrom and Fuerst (2001). Correspondingly, researchers nowadays pay less attention to the role of monetary aggregates and increasingly employ money demand specifications that allow to neglect money for the analysis of equilibrium determination (see Dupor (2001); Woodford (2003) or Carlstrom and Fuerst (2005)). There are two prominent results in this literature (Benhabib et al. 2001a). First, whether an equilibrium allocation is neutral with respect to the absolute price level (nominal indeterminacy) depends exclusively on the stance of fiscal policy and not on monetary policy, preferences or technology. Second, standard assumptions on preferences and technology imply that the Taylor-principle ensures stability and uniqueness of equilibrium sequences if fiscal solvency is guaranteed under all possible circumstances. According to the Taylor-principle (activeness), monetary policy should aggressively fight inflation by raising the nominal interest rate more than the increase in inflation above target.

In this paper, I challenge these prominent findings. I do this by revisiting the role of real balance effects and their timing for equilibrium determination as suggested by Patinkin (1949, 1965). Throughout my analysis I assume that the intertemporal government budget constraint is satisfied under all possible path of endogenous variables. This implies that fiscal policy is not capable to pin down the price level as suggested by the fiscal theory of the price level (see e.g. Leeper 1991; Sims 1994; Woodford 1994, 1995, 1996).

As my main novel result, I show that if the beginning-of-period stock of money facilitates transactions, predetermined real money balances can serve as an endogenous state variable of the economy under interest rate policy. This cardinal function of real money has been so far disregarded in the literature. Notably, linking the provision of transaction services to beginning-of-period instead of end-of period real money balances implies direct costs of inflation for the private sector beyond the indirect costs imposed by reducing wealth – a realistic assumption supported by the literature on the welfare costs of inflation (Chadha, Haldane, and Janssen 1998; Lucas 2000; Palenzuela, Camba-Méndez, and García 2003). In this case, a perfect foresight equilibrium displays nominal determinacy: it is is associated with a unique price level sequence. Key to understand this result is that in the presence of real balance effects optimal intra-temporal substitution between consumption and leisure defines a function that links predetermined beginning-of-period nominal money balances, deflated by current prices, to current consumption and vice versa. This implies that whenever consumption is uniquely determined for all periods (real determinacy) so is the price level. However, real determinacy applies only if real money balances and not just nominal balances are a relevant state variable. But then interest rate policy should rather be passive than active – a violation of the Taylor-principle – to avoid oscillatory or explosive equilibrium sequences, such that a perfect foresight equilibrium is uniquely determined.

The intuition for the failure of the Taylor-principle as a stabilizing device can be explained as follows. Suppose current inflation exceeds its long run equilibrium value. According to the standard reasoning, an active interest rate setting – implying an increase in the expected real interest rate – dampens current consumption, such that this scenario can not be self-fulfilling. However, under real balance effects and when beginning-of-period balances yield transaction services, the expected real interest rate is further negatively related to the growth rate of real balances. Thus an active interest rate setting leads by a standard money demand function to a decline in the level and the growth rate of real balances, such that the sequences of real balances and, thus, of consumption and inflation do not converge to the steady state.

I set up a discrete time general equilibrium model with flexible prices, where real money balances and consumption enter the utility function in a non-separable way, that is consistent with a shopping time technology (McCallum and Goodfriend 1987). I apply two different specifications about the particular stock of money, that enters the utility function: Either the stock of money at the beginning or at the end of the period is assumed to yield transaction services. The idea of the former specification can be interpreted as the money-in-the-utility-function version of Svensson (1985)'s timing of markets within one period, where the goods market is closed, before the asset market is opened. Then, households rely on the stock of money carried over from the previous period for transactions in the goods market implying direct costs of inflation. This formulation is applied for example in Woodford (1990), McCallum and Nelson (1999) or more recently in Persson, Persson, and Svensson (2006). The second specification – when the end-of-period-stock of money yields utility – can be found in Woodford (2003) or Ljungqvist and Sargent (2004). It can be interpreted as a money-in-the-utility-function version for a reverse timing of markets, i.e. households can always adjust their money holdings within one period to facilitate transactions. Notably, only the specification that ties liquidity services to the beginning rather than to the end-of-period real money balances is consistent with direct costs of inflation. The resulting real balance effects are commonly neglected, since they are typically found to be very small (Lucas 2000 or Ireland 2004a). I show, that the existence and the timing of real balance effects (not the magnitude) can have substantial implications for equilibrium determination.

As the main contribution of the paper I find that in the presence of real balance effects and interest rate policy, a uniquely determined price level is associated with real money being a relevant state variable. I.e. with a history dependent evolution of equilibrium sequences. This crucially affects the conditions for macroeconomic stability: an interest rate policy that reacts to changes in current inflation has to be passive for equilibrium sequences to be uniquely determined and to converge to the steady state in a non-oscillatory way. Neither an interest rate peg nor a forward looking interest rate rule lead to this result.

Furthermore I show that when end-of-period money enters the utility function, the equilibrium is characterized by nominal indeterminacy. This result corresponds to the indeterminacy result by Sargent and Wallace (1975). In the perfect foresight equilibrium under interest rate policy only real money balances but not nominal balances and prices can be determined separately. In this case the Taylor-principle ensures local stability and uniqueness of equilibrium sequences. Under a constant money growth regime, a perfect foresight equilibrium displays nominal determinacy, but real money does not serve as a relevant state variable. Equilibrium sequences are, in any case, locally stable and uniquely determined. Remarkably, for the economy to evolve in a history dependent way, it does not suffice, that monetary policy is history dependent.

Related Literature

I now turn to the related literature. Most closely related to my paper is the work by Benhabib et al. (2001a) and Carlstrom and Fuerst (2001) who analyze equilibrium determination under flexible prices. Benhabib et al. (2001a) were among the first to show that conditions for local stability and uniqueness under interest policy are highly sensitive to changes in preferences and technology. However, since they employ a specification that corresponds to my end-of-period formulation they find that nominal determinacy is a purely fiscal phenomenon. Carlstrom and Fuerst (2001) also examine whether the particular stock of money that enters the utility function matters for local stability and uniqueness. They allow for two specifications, either the money stock after agents leave the asset market (which opens first) or the amount of money balances after agents leave the goods market is assumed to enter the utility function. The crucial difference to my approach is that in their model, only the stock of money but not the (implicit) timing of markets changes across the specifications. In particular, since the financial market always opens first, agents do not rely on the stock of money carried over from the previous period to purchase consumption. Correspondingly, predetermined real money balances can not serve as a state variable, and the timing conventions analyzed in Carlstrom and Fuerst (2001) affect only conditions for local stability and uniqueness but not the determination of the absolute price level.

Brückner and Schabert (2005) and Kurozumi (2006) analyze local stability and uniqueness of equilibrium sequences in stochastic maximizing economies under sticky prices, when beginning-of-period money yields transaction services. Employing a shopping-time specification, Brückner and Schabert (2005) too find that interest policy should react passively to changes in inflation to ensure local stability and uniqueness of equilibrium sequences. Kurozumi (2006) studies determinacy and expectational stability of Taylor-rules in a non-separable money-in-the-utility-function framework. He finds that conditions that ensure real determinacy and expectational stability are highly sensitive to assumption on the stock of money that is assumed to deliver transaction services. My contribution is to show that predetermined real money balances play not only a role for learning, local stability and uniqueness of equilibrium sequences under sticky prices. Moreover, predetermined real money balances crucially affect the determination of the absolute price level under flexible prices – a key role for money which has been disregarded in the aforementioned studies.

The remainder of the paper is organized as follows. Section 2 develops the model. Section 3 analyzes nominal and real determinacy under flexible prices. In the first part, I consider the case where the beginning-of-period stock of money provides utility, while the results for the end-of-period specification are briefly summarized in the second part.[1] For both specifications, I derive the implications for equilibrium determination and local stability under current and forward looking interest rate rules, and for money growth rules. The last part of section 3 discusses my findings and compares them to results in related studies. In section 4 I list the main results when prices are set in a staggered way. Section 5 concludes.

4.2 The model

In this section an infinite horizon general equilibrium model with representative agents and perfectly flexible prices is developed. I consider a money in the utility function specification that leads to real balance effects and assume either that the stock of money at the beginning or at the end of the period yields transaction services. Monetary policy is either specified in form of an interest rate feedback rule or constant money growth. To check for the robustness of the results for the former policy regime, I apply contemporaneous and forward looking interest rate rules.

Lower (upper) case letters denote real (nominal) variables. There is a continuum of identical and infinitely lived households. At the beginning of period t, households' financial wealth comprises money M_{t-1} and nominally non-state contingent gov-

[1] My findings for the latter case relate to the results in Benhabib et al. (2001a), Carlstrom and Fuerst (2001), Woodford (2003)

ernment bonds B_{t-1} carried over from the previous period. The households' budget constraint reads

$$M_t + B_t + P_t c_t \leq R_{t-1} B_{t-1} + M_{t-1} + P_t w_t l_t - P_t \tau_t, \tag{4.1}$$

c_t denotes consumption, P_t the aggregate price level, w_t the real wage rate, l_t working time, τ_t a lump-sum tax, and R_t the gross nominal interest rate on government bonds. Further, households have to fulfill the no-Ponzi game condition, $\lim_{t\to\infty}(m_t + b_t)\prod_{i=1}^{t} \pi_i / R_{i-1} \geq 0$, where $b_t = B_t / P_t$ and $m_t = M_t / P_t$ denote real bonds and real money balances. The objective of the representative household is

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t, A_t / P_t\right), \quad \beta \in (0,1), \tag{4.2}$$

β denotes the subjective discount factor and A_t nominal balances, which will be defined below. The instantaneous utility function is assumed to satisfy

$$u_c > 0,\, u_l < 0,\ u_a > 0,\, u_{cc} < 0,\ u_{aa} < 0,\ u_{ll} \leq 0, \tag{4.3}$$

$$u_{ca} > 0,\ u_{cl} = u_{al} = 0,\ u_{cc} u_{aa} - u_{ca}^2 > 0, \tag{4.4}$$

and the usual Inada-conditions, where $a_t = A_t / P_t$. According to (4.4) the cross derivative u_{ca} is (strictly) positive, such that marginal utility of consumption rises with real money balances. The resulting properties, i.e., non-separability and real balance effects, typically emerge under more explicit specifications of transaction frictions. As, for example, shown by Brock (1974) or Feenstra (1986), a money-in-the-utility (MIU) function specification, which is equivalent to a specification where purchases of consumption goods are associated with transaction costs that are either measured by shopping time or real resources, is usually characterized by these properties. To be more precise, introducing these transaction frictions in a corresponding model with a utility function $v(c_t, 1 - l_t)$ would lead to real balance effects, which are equivalent to a MIU specification with $u_{ca} > 0$, if (but not only if) the labor supply elasticity is finite (see appendix 5.3.1). It should be noted that an infinite labor supply elasticity will lead to be of particular interest in what follows.

To avoid additional complexities, I assume that the respective cross derivatives are equal to zero $u_{lc} = u_{la} = 0$.[2] The last assumption in (4.4), $u_{cc} u_{aa} - u_{ca}^2 > 0$, is imposed to ensure – together with (4.3) – the utility function to be strictly concave. The conditions in (4.3)-(4.4) further ensure that real money balances and consumption

[2] This implies that the instantaneous utility function $u(c_t, a_t, l_t)$ can be written as $f(c_t, a_t) - g(l_t)$.

are normal goods, i.e. that the utility function exhibits increasing expansion paths with respect to money and consumption.

The variable A_t describes the relevant stock of money that provides – in real terms – utility. Throughout the paper, I consider two cases, where A_t denotes money either held at the **B**eginning of the period, M_{t-1}, or at the **E**nd of period, M_t :

$$A_t = \begin{cases} M_{t-1} & \textbf{B}\text{-version} \\ M_t & \textbf{E}\text{-version} \end{cases} .$$

The **B**-version, which, for example, relates to the money-in-the-utility function specifications in McCallum and Nelson (1999); Woodford (1990), is more recently applied in Persson, Persson, and Svensson (2006). It can be motivated as the money-in-the-utility-function version of Svensson (1985)'s timing of markets assumption within one period where the goods market is closed before the asset market is opened. This case is illustrated in 4.1. It means that the representative agent in period t relies on the stock of money carried over from the previous period M_{t-1} for transactions in the goods market – implying that a surprise inflation immediately affects households' utility. After the goods market is closed, households adjust their nominal balances on the asset market according to their planned consumption expenditures in $t+1$.

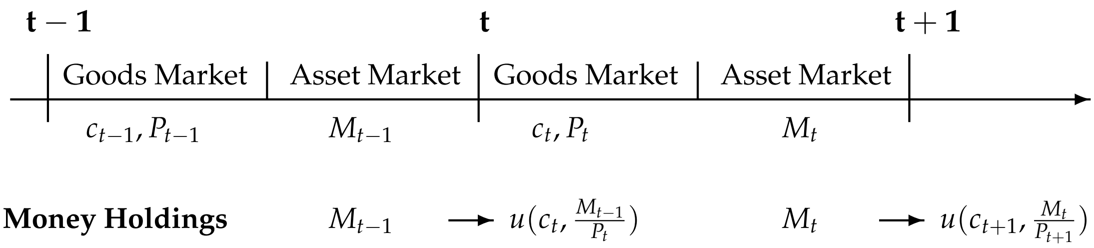

Figure 4.1: Timing of markets under beginning-of-period money (**B**-version)

On the contrary, in the end-of-period specification (**E**-version), which can for example be found in Brock (1974); Ljungqvist and Sargent (2004), or Woodford (2003), the stock of money held at the end of the period yields utility. This formulation can be interpreted as the money-in-the-utility function version of a reverse timing of markets, i.e. the asset market is closed before the goods market is opened. In this case, which is illustrated in figure 4.2, agents can freely adjust their nominal balances to purchase consumption goods within period t. This implies that households are less prone to be harmed by surprise inflation.

Maximizing (4.2) subject to (4.1) and the no-Ponzi game condition for given initial

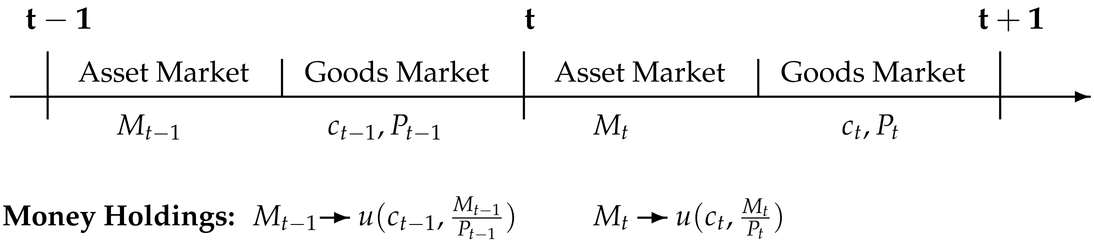

Figure 4.2: Timing of markets under end-of-period money (**E**-version)

values $M_{-1} > 0$ and $R_{-1}B_{-1} \geq 0$ leads to the following first order conditions for consumption, money, labor supply, and government bonds:

$$\lambda_t = \begin{cases} u_c\left(c_t, m_{t-1}/\pi_t\right) & \textbf{B}\text{-version} \\ u_c\left(c_t, m_t\right) & \textbf{E}\text{-version} \end{cases}, \tag{4.5}$$

$$i_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \begin{cases} u_a\left(c_{t+1}, m_t/\pi_{t+1}\right)/\pi_{t+1} & \textbf{B}\text{-version} \\ \beta^{-1} u_a\left(c_t, m_t\right) & \textbf{E}\text{-version} \end{cases}, \tag{4.6}$$

$$u_l(l_t) = -w_t \lambda_t, \tag{4.7}$$

$$\lambda_t = \beta R_t \lambda_{t+1} \pi_{t+1}^{-1}, \tag{4.8}$$

where $i_t = R_t - 1$ denotes the net interest rate on government bonds, λ_t denotes a Lagrange multiplier, π_t the inflation rate $\pi_t = P_t/P_{t-1}$. Note that beginning-of-period real balances m_{t-1} enter the set of first order conditions only in the **B**-version and only together with the current inflation rate – such that alternatively one could have written the conditions in terms of M_{t-1}/P_t. Thus, in principle, both versions are forward-looking. Nevertheless, I will show below that beginning-of-period real money balances can restrict current consumption, if they serve as a relevant state variable. The optimum is further characterized by the budget constraint (4.1) holding with equality and by the transversality condition $\lim_{t\to\infty}(m_t + b_t)\prod_{i=1}^{t} \pi_i/R_{i-1} = 0$.

There is a continuum of perfectly competitive firms of mass one. Firms produce the consumption good c_t with the linear technology $y_t = l_l$. The only production factor labor, supplied by households, is hired on a competitive labor market – implying that profit maximization leads to zero profits and a real wage w_t of unity. Total output comprises private consumption.

The public sector consists of a fiscal and a monetary authority. I consider two widely applied specifications for the monetary policy regime. The first regime is characterized by the central bank setting the nominal interest rate contingent on current or on future inflation.

$$R_t = \rho\left(\pi_t\right), \quad \text{or} \quad R_t = \rho\left(\pi_{t+1}\right), \qquad \text{with } \rho' \geq 0, \quad R_t \geq 1. \tag{4.9}$$

I further assume that the steady state condition $R = \pi/\beta$ has a unique solution for $R > 1$. According to the interest rate feedback rule (4.9), the response of the interest rate to changes in inflation, ρ_π, is non-negative. The second regime, is characterized by the central bank holding the money growth constant:

$$m_t\pi_t/m_{t-1} = \mu, \quad \mu \geq 1. \tag{4.10}$$

The fiscal authority issues risk-free one period bonds, receives lump-sum taxes from households, and transfers from the monetary authority. I assume that tax policy guarantees government solvency (Ricardian fiscal policy), i.e. it ensures that $\lim_{t\to\infty}(m_t + b_t)\prod_{i=1}^{t}\pi_i/R_{i-1} = 0$.

4.3 Equilibrium determination under flexible prices

In this section, I assess how real balance effects, the timing of markets and monetary policy affect the determination of the price level and of the perfect foresight equilibrium. As described in the previous section, I consider two versions of the model which differ with regard to the stock of money that enters the utility function, i.e., the **B**-version and the **E**-version, and I consider three types of monetary policy rules described by (4.9) or (4.10). The equilibrium for a positive interest rate ($R_t > 1$) for both versions can then be summarized as follows.

Definition 4.1. *Given an initial money endowment M_{-1}, a Ricardian fiscal policy $\tau_t \forall t \geq 0$ and a monetary policy (4.9) or (4.10), a perfect foresight equilibrium (PFE) consists of set of sequences $\{c_t,\ \pi_t,\ m_t,\ R_t\}_{t=0}^{\infty}$ and a price level P_0, satisfying $\forall t \geq 0$ the transversality condition $\lim_{t\to\infty}(m_t + b_t)\prod_{i=1}^{t}\pi_i/R_{i-1} = 0$, and either $u_l(c_t) = -u_c\left(c_t, \frac{m_{t-1}}{\pi_t}\right)$, $u_c\left(c_t, \frac{m_{t-1}}{\pi_t}\right) = \beta R_t u_c\left(c_{t+1}, \frac{m_t}{\pi_{t+1}}\right)/\pi_{t+1}$, and $(R_t - 1)u_c\left(c_{t+1}, \frac{m_t}{\pi_{t+1}}\right) = u_a\left(c_{t+1}, \frac{m_t}{\pi_{t+1}}\right)$ for the **B**-version or $u_l(c_t) = -u_c\left(c_t, m_t\right)$, $u_c\left(c_t, m_t\right) = \beta R_t u_c\left(c_{t+1}, m_{t+1}\right)/\pi_{t+1}$, and $(R_t - 1)u_c\left(c_{t+1}, m_{t+1}\right)/\pi_{t+1} = u_a\left(c_t, m_t\right)/\beta$ for the the **E**-version.*

Notably, in contrast to initial nominal balances M_{-1}, which shows up in the condition for optimal intra-temporal substitution $u_l(c_0) = -u_c\left(c_0, \frac{M_{-1}}{P_0}\right)$ for $t = 0$ in the **B**-version, there is no need to include an initial price level P_{-1} in the set of relevant state variable under flexible prices: neither resources nor the optimal actions of private agents or the government depend on the initial price level. This implies that initial nominal balances and not initial real balances m_{-1} are the relevant state variable.

The dependence of a given allocation on a particular absolute price level in the first period P_0 is often summarized by the notion "nominal determinacy"İt is crucial to note that the role of the price level in the first period does not relate to the unique determination of equilibrium sequences (including the inflation sequence) which is summarized by the notion "real determinacy". These properties are summarized in the following Definition, which corresponds to the Definition applied in Benhabib et al. (2001a).

Definition 4.2. *The equilibrium displays real determinacy if there exists a unique set of equilibrium sequences* $\{c_t, \pi_{t+1}, m_t, R_t\}_{t=0}^{\infty}$. *Given* M_{-1}, *the equilibrium displays nominal indeterminacy if for any particular set of equilibrium sequences* $\{c_t, \pi_{t+1}\}_{t=0}^{\infty}$, *there exist infinite many price levels* P_0 *consistent with a perfect foresight equilibrium.*

In the following Proposition I show that the two version, **B** and **E**, differ substantially with respect the determinacy of the price level under interest rate policy.

Proposition 4.3. *Consider that consumption and real money balances enter non-separably into the utility function and that monetary policy targets the nominal interest rate according to (4.9). Then the equilibrium displays nominal determinacy in the* **B***-version and nominal indeterminacy in the* **E***-version.*

Proof. In the **B**-version, for a given sequence $\{c_t\}_{t=0}^{\infty}$, if $u_{ca} \neq 0$, the condition for optimal intra-temporal substitution between consumption and leisure, $u_l(c_t) = -u_c\left(c_t, \frac{m_{t-1}}{\pi_t}\right)$, defines implicitly a monotone function $M_{t-1}/P_t = f(c_t)$ for all $t \geq 0$. Thus a given sequence $\{c_t\}_{t=0}^{\infty}$ results in an unique sequence $\{M_{t-1}/P_t\}_{t=0}^{\infty}$. Given M_{-1}, P_0 is uniquely determined. In the **E**-version and interest policy, the equilibrium conditions determine just real money balances m_t, and do not disentangle real money balances m_0 into its components M_0 and P_0. It follows that the perfect foresight equilibrium is consistent with infinitely many pairs P_0, M_0, and the equilibrium displays nominal indeterminacy. □

If consumption and real money balances enter non-separably into the utility function and the **B**-version applies, real determinacy is sufficient for the determination of

P_0, such that nominal determinacy applies. However, in the **E**-version, the equilibrium under interest policy displays nominal indeterminacy even if the equilibrium is characterized by real determinacy.[3] The latter results corresponds to the main result in Sargent and Wallace (1975). Since in the **E**-version there is an infinite number of equilibrium pairs for nominal money balances and the price level, sunspot equilibria may occur. If the assumed welfare measure punishes fluctuations in prices, Sargent and Wallace conclude that monetary policy should not target interest rates but monetary aggregates. I show that their conclusion depends on the implicit timing of markets: it applies only for the **E**-version, when end of period is assumed to yield transaction services, but not in the **B**-version when beginning of period money delivers utility.

A PFE, which is characterized by real determinacy and, thus, a unique inflation sequence, can be associated with multiple price level sequences, even if beginning-of-period money enters the utility function. If there are no real balance effects ($u_{ca} = 0$), real money balances are determined residually by the forward-looking money demand equation $(R_t - 1)u_c(c_{t+1}) = u_a\left(\frac{m_t}{\pi_{t+1}}\right)$ without any relation to initial nominal balances. This implies that nominal balances and the price level can not be determined separably, and the price level is neutral with regard to the determination of equilibrium sequences $\{c_t, \pi_t, R_t\}_{t=0}^{\infty}$ under interest rate policy. Thus, two different values for the initial price level together with an equilibrium inflation sequence lead to two different price level sequences consistent with the PFE. Evidently, one cannot uniquely determine a unique price level sequence if there are infinitely many equilibrium inflation sequences.

In the next Proposition I show that the equilibrium under a constant money growth rate is associated with a particular price level in the first period for both versions.

Proposition 4.4. *Under a constant money growth rule the equilibrium displays nominal determinacy for both versions* **B** *and* **E**.

Proof. Given M_{-1}, a constant money growth rule uniquely pins down a whole sequence for nominal balances $\{M_t\}_{t=0}^{\infty}$. Given a sequence $\{m_t\}_{t=0}^{\infty}$, the whole sequence for the price level $\{P_t\}_{t=0}^{\infty}$ is uniquely determined. □

Independent of the existence of real balance effects, the PFE under a constant money growth rule is associated with a unique price level sequence, whenever $\{m_t\}_{t=0}^{\infty}$ is uniquely determined.

[3] By construction, the equilibrium in the **E**-version would be nominally determinate, if the interest rate policy were to react exclusively on the current price level.

To summarize, under interest rate policy and if there are no real balance effects, the equilibrium displays nominal indeterminacy in both versions. Given real determinacy and the presence of real balance effects, the equilibrium in the **B**-version exhibits nominal determinacy, while in the **E**-version the equilibrium is in any case associated with multiple price level sequences under interest rate policy. Under a constant money growth policy, the equilibrium is characterized by nominal determinacy if real determinacy applies. Whether real determinacy is ensured or not depends on monetary policy.

However, the exact conditions under which real determinacy is ensured depend crucially on the stance of monetary policy and will be the focus of the following analysis. I apply Blanchard and Kahn's (1980) approach to the analysis of a perfect foresight equilibrium. For this, I focus on the model's behavior in the neighborhood of the steady state, and apply a linear approximation of the set of non-linear equilibrium conditions. Throughout, I restrict my attention to *locally stable* equilibrium sequences, i.e., to equilibrium sequences that converge to the steady state.

4.3.1 Beginning-of-period money

I start with the case where the beginning-of-period stock of money enters the utility function. The deterministic steady state is then characterized by the following properties: $\overline{R} = \overline{\pi}/\beta$, $-u_l(\overline{c}) = u_c(\overline{c}, \overline{m}/\overline{\pi})$, and $u_c(\overline{c}, \overline{m}/\overline{\pi})\left(\overline{R}-1\right) = u_a(\overline{m}/\overline{\pi}, \overline{c})$. A discussion of the existence and uniqueness of a steady state for $\overline{R} > 1$ can be found in appendix 5.3.2 . Log-linearizing the model at the steady state, leads to the following set of equilibrium conditions:

$$\varepsilon_{ca}\widehat{m}_{t-1} - \varepsilon_{ca}\widehat{\pi}_t = (\sigma_l + \sigma_c)\,\widehat{c}_t, \tag{4.11}$$

$$\sigma_c\widehat{c}_t - \varepsilon_{ca}\widehat{m}_{t-1} + \varepsilon_{ca}\widehat{\pi}_t = \sigma_c\widehat{c}_{t+1} - \varepsilon_{ca}\widehat{m}_t + (\varepsilon_{ca}+1)\,\widehat{\pi}_{t+1} - \widehat{R}_t, \tag{4.12}$$

$$(\varepsilon_{ca} + \sigma_a)\,\widehat{m}_t = -z\widehat{R}_t + (\sigma_c + \phi_{ac})\,\widehat{c}_{t+1} + (\varepsilon_{ca} + \sigma_a)\,\widehat{\pi}_{t+1}, \tag{4.13}$$

where $z \equiv \overline{R}/(\overline{R}-1) > 1$, $\sigma_l \equiv \frac{\overline{l}\overline{u}_{ll}}{\overline{u}_l} \geq 0$, $\sigma_c \equiv -\frac{\overline{c}\overline{u}_{cc}}{\overline{u}_c} > 0$, $\sigma_a \equiv -\frac{\overline{a}\overline{u}_{aa}}{\overline{u}_a} > 0$, $\varepsilon_{ca} \equiv \frac{\overline{a}\overline{u}_{ca}}{\overline{u}_c} > 0$, and $\phi_{ac} \equiv \frac{\overline{c}\overline{u}_{ac}}{\overline{u}_a} > 0$, and $\widehat{f}_t$ denotes the percent deviation of a generic variable f_t from its steady state value $\overline{f}$: $\widehat{f}_t = \log(f_t) - \log(\overline{f})$. These conditions (and the transversality condition) have to be satisfied by the equilibrium sequences for the steady state deviations of consumption, real balances, the inflation rate, and of the nominal interest rate, $\{\widehat{c}_t, \widehat{\pi}_t, \widehat{m}_t, \widehat{R}_t\}_{t=0}^{\infty}$ and a monetary policy regime satisfying

$$\widehat{R}_t = \rho_\pi \widehat{\pi}_t, \quad \text{or} \quad \widehat{R}_t = \rho_\pi \widehat{\pi}_{t+1}, \quad \text{or} \quad \widehat{m}_t = \widehat{m}_{t-1} - \widehat{\pi}_t, \tag{4.14}$$

where ρ_π denotes the steady state inflation elasticity $\rho_\pi \equiv \rho'(\overline{\pi})(\overline{\pi}/\overline{R}) \geq 0$. Following Benhabib et al. (2001a), interest rate policy is called *active* or according to the Taylor-principle if $\rho_\pi > 1$, and *passive* if $\rho_\pi < 1$. An active (passive) interest rate setting leads to an increase (decrease) in the real interest rate in response to an increase in the inflation measure. It should be noted that concavity of the utility function implies: $\Upsilon \equiv \sigma_c \sigma_a - \varepsilon_{ca} \phi_{ac} > 0$,[4] which restricts the magnitude of real balance effects. A closer look at the equilibrium conditions (4.11) and (4.12) reveals that the private sector behavior is not independent of the beginning-of-period value for real balances $\widehat{m}_{t-1}$, as they are (implicitly) assumed to lower households' transactions costs. Given that $\widehat{m}_{t-1}$ is predetermined, the households' behavior can induce the economy to evolve in a history dependent way, i.e. predetermined real money balances can be a state variable. Defining $[\widehat{m}_t, \widehat{c}_t, \widehat{\pi}_t, \widehat{R}_t]' \equiv \widehat{x}_t$, the following Definition summarizes this property.

Definition 4.5. *Consider the fundamental solution for the equilibrium sequences $\{\widehat{x}_t\}_{t=0}^\infty$, that satisfies the equilibrium conditions (4.11)-(4.13) and one monetary policy rule (4.14). If there exists a locally stable and unique fundamental solution of the linear functional form*

$$\widehat{x}_t = \begin{pmatrix} \eta_m \\ \eta_c \\ \eta_\pi \\ \eta_R \end{pmatrix} \widehat{m}_{t-1} = \Lambda \widehat{m}_{t-1} \quad \forall t \geq 0 \tag{4.15}$$

with $\eta_i \neq 0$ for $i = m, c, \pi, R$, then predetermined real money balances are an endogenous state variable.

It is crucial to note, that if real money balances are a state variable, then not only first period values $\widehat{x}_0$ are associated with a particular first period price level. Instead, the whole set of equilibrium variables is indexed with a specific value for m_{-1} a at each point in time, i.e. $\widehat{x}_t = \Lambda.^{t+1} \widehat{m}_{-1}$, $\forall t > 0$. For a given initial value M_{-1}, the set of equilibrium sequences relies on a particular initial price level P_{-1}. Since this mechanism applies to each period, the complete set of sequences for the absolute price level $\{P_t\}_{t=0}^\infty$ and nominal balances $\{M_t\}_{t=0}^\infty$ is uniquely determined. Evidently, if real money balances are a state variable, the equilibrium displays nominal determinacy. But as will become clear below, the reverse must not be true.

[4] I view this as a realistic implication, given that estimates of ε_{ca} and ϕ_{ac}, are usually found to be small. According to US estimates reported in Woodford (2003), ε_{ca} does not exceed 0.005 and $\phi_{ac} \leq 2$.

Yet, $\widehat{m}_{t-1}$ enters the equilibrium conditions jointly with the current inflation rate. Thus, predetermined real money serves as a relevant endogenous state variable, only if the current inflation $\widehat{\pi}_t$ rate is uniquely determined, which implies real determinacy. Given real determinacy, nominal determinacy applies, whenever the beginning-of-period stock of money enters the utility function. But monetary policy is decisive for real determinacy, i.e. for the possibility to uniquely determine a price level sequence. In the subsequent analysis, I will show that this requires the central bank to set the nominal interest rate contingent on current inflation. Under an interest rate peg, $\rho_\pi = 0$, an inflation sequence and, therefore, a price level sequence cannot be uniquely determined.[5]

In the following Proposition I establish the main novel result that real money balances serve as a state variable when current inflation is the policy indicator under interest rate policy, and the labor supply elasticity $1/\sigma_l$ is finite.

Proposition 4.6. *Consider that beginning-of-period money enters the utility function and that the nominal interest rate is set contingent on changes in current inflation* $\widehat{R}_t = \rho_\pi \widehat{\pi}_t$.

1. *When the labor supply elasticity is finite, $\sigma_l > 0$, predetermined real money balances serve as a state variable. The equilibrium displays real determinacy and local stability if and only if*
   (a) $\overline{\rho_{\pi 1}} < \rho_\pi < 1$ *for* $\varepsilon_{ca} > \frac{\sigma_a}{2z-1}$ *and* $\sigma_l > \overline{\sigma_l}$, *leading to non-oscillatory equilibrium sequences, or* $\rho_\pi \in (1, \overline{\rho_{\pi 1}})$, *leading to oscillatory equilibrium sequences,*
   (b) $\rho_\pi > 1$ *for* $\varepsilon_{ca} < \frac{\sigma_a}{2z-1}$ *or* $\sigma_l < \overline{\sigma_l}$, *leading to oscillatory equilibrium sequences,*

   where $\overline{\rho_{\pi 1}} \equiv \frac{\sigma_l(\varepsilon_{ca}+\sigma_a)+Y}{\sigma_l(2z-1)\varepsilon_{ca}-\sigma_l\sigma_a-Y}$ *and* $\overline{\sigma_l} \equiv \frac{Y}{(2z-1)\varepsilon_{ca}-\sigma_a}$.
2. *When the labor supply elasticity is infinite, $\sigma_l = 0$, predetermined real money balances do not serve as a state variable. Consumption $\widehat{c}_t$ cannot uniquely be determined, while the equilibrium sequences $\{\widehat{c}_{t+1}, \widehat{\pi}_t, \widehat{m}_t, \widehat{R}_t\}_{t=0}^{\infty}$ are locally stable and uniquely determined if and only if $\rho_\pi > 1$.*

Proof. See appendix 5.3.3.

Proposition 4.6 reveals that the requirements for local equilibrium stability and uniqueness in terms of the policy parameter ρ_π are not robust with regard to changes

[5]It should further be noted that a *PFE* displays nominal indeterminacy if there are no real balance effects, $\varepsilon_{ca} = \phi_{ac} = 0$. Nevertheless, one can always compute a price level sequence for a particular initial price level and a sequence of inflation rates.

in the elasticities ε_{ca} and σ_l.[6] For finite labor supply elasticities, $\sigma_l > 0$, predetermined real balances serve as an endogenous state variable. Correspondingly, passiveness ($\rho_\pi < 1$) – a violation of the Taylor-principle – is necessary for locally stable, unique, and non-oscillatory equilibrium sequences (see part 1a). An interest rate peg, however, violates the conditions in part 1 of Proposition 4.6 and, thus, implies real indeterminacy. On the contrary, if interest rate policy follows the Taylor-principle ($\rho_\pi > 1$), locally stable and unique equilibrium sequences are oscillatory, which is hardly recommendable for a central bank that aims at stabilizing the economy. Thus, when beginning-of-period money relates to households' consumption, interest rate policy that reacts on current inflation should rather be passive than active for macroeconomic stability and for the unique determination of the price level.

To see this, suppose that inflation exceeds its steady state value and equilibrium sequences are non-oscillatory.[7] Given that the inflation elasticity is positive, $\rho_\pi > 0$, the nominal interest rate rises, which – ceteris paribus – causes households to reduce their end-of-period real money holdings $\widehat{m}_t$, by (4.13). According to (4.12), the expected real interest rate is further negatively related to the growth rate of real balances. Thus, an active interest rate setting – implying an increase in the real interest rate – leads to a decline in the level and the growth rate of real balances, such that the sequences of real balances and, thus, of consumption and inflation do not converge to the steady state.

Notably, the equilibrium exhibits different properties if the marginal disutility of labor is constant, i.e. if the inverse of the labor supply elasticity is zero (see part 2 of Proposition 4.6). Then, the amount of labor supplied by the households is not related to their consumption expenditures and and the marginal utility of consumption is always identical to its steady state value (see 4.11). In this case, the Euler equation and money demand reduce to a constant real interest rate $\widehat{R}_t - \widehat{\pi}_{t+1} = 0$, and $\sigma_a \widehat{m}_t = -z\widehat{R}_t + \phi_{ac}\widehat{c}_{t+1} + \sigma_a \widehat{\pi}_{t+1}$, such that the equilibrium is not associated with a unique value for beginning-of-period real money and that current consumption $\widehat{c}_t$ cannot be determined. Correspondingly, predetermined real money balances do not serve as a state variable. The equilibrium sequences for $\widehat{c}_{t+1}$, $\widehat{\pi}_t$, $\widehat{m}_t$, and $\widehat{R}_t$ are then locally stable and uniquely determined for an active interest rate policy, which contrasts the results for the case of finite labor supply elasticities, $\sigma_l > 0$, presented in part 1 of Proposition 4.6.

[6]Note that for the sets $(\overline{\rho_{\pi 1}}, 1)$ and $(1, \overline{\rho_{\pi 1}})$ (see part 1a. of Proposition 4.6) to be non-empty $\sigma_l > \Upsilon[(z-1)\varepsilon_{ca} - \sigma_a]^{-1}$ and $\varepsilon_{ca} > \sigma_a/(z-1)$, and, respectively, $\sigma_l < \Upsilon[(z-1)\varepsilon_{ca} - \sigma_a]^{-1}$ or $\varepsilon_{ca} < \sigma_a/(z-1)$ has to be satisfied.

[7]The latter property implies that current and expected future inflation are not negatively related.

I now turn to the case where the central bank applies a forward looking rule, $\widehat{R}_t = \rho_\pi \widehat{\pi}_{t+1}$.

Proposition 4.7. *Consider that beginning-of-period money enters the utility function and that the nominal interest rate is set contingent on changes in future inflation $\widehat{R}_t = \rho_\pi \widehat{\pi}_{t+1}$. Then, consumption and inflation cannot uniquely be determined and predetermined real money balances do not serve as a state variable.*

1. *When the labor supply elasticity is finite, $\sigma_l > 0$, then $\rho_\pi > 1$ is a necessary condition for uniqueness and local stability of the equilibrium sequences $\{\widehat{c}_{t+1}, \widehat{\pi}_{t+1}, \widehat{m}_t, \widehat{R}_t\}_{t=0}^{\infty}$.*
   Necessary and sufficient conditions are given by:

   (a) $1 < \rho_\pi$ *for* $\sigma_l > \overline{\sigma_{l2}}$ *and* $\varepsilon_{ca} > \frac{\sigma_a}{z-1}$,

   (b) $1 < \rho_\pi < \overline{\rho_{\pi 2}}$, *for* $\sigma_l < \overline{\sigma_{l2}}$ *or* $\varepsilon_{ca} < \frac{\sigma_a}{z-1}$, *or* $1 < \overline{\rho_{\pi 2}} < \rho_\pi$ *if* $\sigma_l > \overline{\sigma_l}$ *and* $\varepsilon_{ca} > \frac{\sigma_a}{2z-1}$, *for* $\sigma_l \in (\overline{\sigma_l}, \overline{\sigma_{l2}})$ *or* $\varepsilon_{ca} \in (\frac{\sigma_a}{2z-1}, \frac{\sigma_a}{z-1})$,

   (c) $1 < \overline{\rho_{\pi 2}} < \rho_\pi < -\overline{\rho_{\pi 1}}$ *for* $\sigma_l < \overline{\sigma_l}$ *or* $\varepsilon_{ca} < \frac{\sigma_a}{2z-1}$,

   where $\overline{\sigma_{l2}} \equiv \frac{\Upsilon}{(z-1)\varepsilon_{ca}-\sigma_a}$ *and* $\overline{\rho_{\pi 2}} \equiv \frac{\Upsilon+\sigma_l(\varepsilon_{ca}+\sigma_a)}{\Upsilon+\sigma_l(\varepsilon_{ca}+\sigma_a)-z\varepsilon_{ca}\sigma_l}$.

2. *When the labor supply elasticity is infinite, $\sigma_l = 0$, then the equilibrium sequences $\{\widehat{c}_{t+1}, \widehat{\pi}_{t+1}, \widehat{m}_t, \widehat{R}_t\}_{t=0}^{\infty}$ are locally stable and uniquely determined if and only if $\rho_\pi \neq 1$.*

Proof. See appendix 5.3.4.

In comparison to Proposition 4.6 the most fundamental difference relates to the role of beginning-of-period real balances, $\widehat{m}_{t-1}$. If monetary policy applies a forward looking interest rate rule, households' optimal consumption decisions are not affected by predetermined real money balances. I.e. real money balances are not a state variable of the economy. The initial stock of real money balances $m_{-1} = M_{-1}/P_{-1}$ is irrelevant for the equilibrium allocation and thus, there are multiple price level sequences. Correspondingly, current inflation can not be pinned down since it enters jointly with $\widehat{m}_{t-1}$ and the equilibrium is consistent with infinitely many values for current inflation. Given that the current values for inflation and consumption can not be determined, households adjust $\widehat{m}_t$ in accordance with their planned future consumption $\widehat{c}_{t+1}$, implying that their behavior is not history dependent. On the contrary, if current inflation serves as a policy indicator, predetermined real money balances restrict households' consumption decisions and initial real money balances are relevant for the equilibrium sequences $\widehat{c}_t, \widehat{\pi}_t, \widehat{m}_t, \widehat{R}_t$ at each point in time: predetermined real money balances are an endogenous state variable (see Definition 3) and the perfect foresight equilibrium is characterized by nominal determinacy. Remarkably in that case, by applying an interest rate rule, the complete set of nominal sequences, the absolute price level and nominal balances, can be uniquely determined.

Under an interest rate rule featuring current inflation, it turns out that there is no robust value for the inflation elasticity that ensures local stability and uniqueness. For example, when the real balance effect and the labor supply elasticity satisfy $\varepsilon_{ca} > \frac{\sigma_a}{2z-1}$ and $\sigma_l > \frac{\Upsilon}{(2z-1)\varepsilon_{ca}-\sigma_a}$, interest rate policy should be passive, $\rho_\pi < 1$, while the inverse, $\rho_\pi > 1$, is required under $\varepsilon_{ca} < \frac{\sigma_a}{2z-1}$ or $\sigma_l < \overline{\sigma_l}$ (see Proposition 4.6). When the central bank sets the nominal interest rate contingent on expected future inflation, activeness $\rho_\pi > 1$ is always necessary (but not sufficient) for uniqueness.[8] As in the previous case (see part 2 of Proposition 4.6), the equilibrium exhibits different properties if the labor supply elasticity is infinite $\sigma_l = 0$ as described in part 2 of Proposition 4.7. With a forward looking interest rate rule, the model then reduces to a set of static equilibrium conditions characterized by unique equilibrium sequences $\{\widehat{c}_{t+1}, \widehat{\pi}_{t+1}, \widehat{m}_t, \widehat{R}_t\}_{t=0}^{\infty}$ for any non-zero inflation elasticity $\rho_\pi \neq 1$.

Under a money growth regime equilibrium determination is less sensitive. Ruling out unreasonable parameter values, I focus, for convenience, on the case where the inverse of the elasticity of intertemporal substitution of money is not extremely large, $\sigma_a < z = \overline{R}/(\overline{R}-1)$.[9]

Proposition 4.8. *Suppose that beginning-of-period money enters the utility function and that $\sigma_a < z$. Under a constant money growth rule, predetermined real money balances do not serve as a state variable. The equilibrium sequences $\{\widehat{c}_{t+1}, \widehat{\pi}_{t+1}, \widehat{m}_t, \widehat{R}_t\}$ $\forall t \geq 0$ are locally stable and uniquely determined, and there exists a unique consistent price level $\forall t \geq 0$.*

Proof. See appendix 5.3.5.

A comparison of the results in the propositions 4.6-4.8 shows that the *PFE* displays real determinacy, if and only if predetermined real money balances are an endogenous state variable. This requires an interest policy contingent on current inflation.

[8]Non-emptiness of the sets for ρ_π requires $\overline{\rho_{\pi 2}} > 1$ and $-\overline{\rho_{\pi 1}} > \overline{\rho_{\pi 2}}$, which is fulfilled for the given restrictions on σ_l and ε_{ca} in part 1b and 1c.

[9]It should be noted that $\sigma_a < z$ is just a sufficient precondition for the result in Proposition 4.8 and hardly restrictive if one assigns values for σ_a that relate to reasonable magnitudes of σ_c.

Remarkably, the money growth regime leads to an equilibrium behavior being different from the behavior under both interest rate policy regimes. On the one hand, the price level can always be determined if real balances are determined, given that the value for the nominal stock of money is known in every period. On the other hand, the initial values for the inflation rate $\widehat{\pi}_0$ and real money $\widehat{m}_{-1}$ are irrelevant for equilibrium determination, implying that there are – for different initial price levels – multiple values for both which are consistent with a unique set of equilibrium sequences $\{\widehat{c}_{t+1}, \widehat{\pi}_{t+1}, \widehat{m}_t, \widehat{R}_t\}_{t=0}^{\infty}$. I.e. the equilibrium displays nominal determinacy but does not rely on predetermined real money balances as an endogenous state variable. Put differently, for the economy to evolve in a history dependent way, it is, therefore, not sufficient that monetary policy is conducted in a backward looking way. Instead, it is the households' consumption decision rather than a restriction on the evolution of money, which is responsible for the equilibrium sequences to depend on beginning-of-period money holdings. There is an analogy to the role of physical capital in a standard real business cycle model with a depreciation rate equal to one. Capital remains a relevant state variable, even though the model (virtually) lacks an accumulation equation.[10]

4.3.2 End-of-period money

Next, I will briefly summarize the requirements for equilibrium determination under the assumption that end-of-period money holdings enter the utility function. In this case, the equilibrium displays nominal indeterminacy – unless monetary policy follows a constant money growth rule. This specification has also been examined by Benhabib et al. (2001a) and by Woodford (2003) for interest rate policies, and by Carlstrom and Fuerst (2003) for money growth rules. The deterministic steady state for this version is characterized by the following conditions, $\overline{R} = \overline{\pi}/\beta$, $-u_l(\overline{c}) = u_c(\overline{c}, \overline{m})$, and $u_c(\overline{c}, \overline{m})\,(R-1) = u_a(\overline{m}, \overline{c})$.[11] Log-linearizing the model summarized in Definition 1 for $A_t = M_t$ at the steady state with $\overline{R} > 1$ leads to the

[10]Consider a real version of my model, with perfect competition, a production technology satisfying $y_t = s_t k_{t-1}^{\alpha} l_t^{1-\alpha}$, where k_{t-1} denotes the beginning-of-period stock of physical capital and $\alpha \in (0,1)$, and a capital depreciation rate of 100%. Nevertheless, capital serves as a relevant state variable, i.e., k_{t-1} affects the equilibrium allocation in period t.

[11]A discussion of steady state uniqueness is provided in appendix 5.3.2.

following set of equilibrium conditions:

$$\varepsilon_{ca}\widehat{m}_t = (\sigma_l + \sigma_c)\,\widehat{c}_t, \tag{4.16}$$

$$\sigma_c\widehat{c}_t - \varepsilon_{ca}\widehat{m}_t = \sigma_c\widehat{c}_{t+1} - \varepsilon_{ca}\widehat{m}_{t+1} - \widehat{R}_t + \widehat{\pi}_{t+1}, \tag{4.17}$$

$$(\varepsilon_{ca} + \sigma_a)\,\widehat{m}_t = (\phi_{ac} + \sigma_c)\,\widehat{c}_t - (z-1)\,\widehat{R}_t. \tag{4.18}$$

The conditions (4.16)-(4.18), the transversality condition, and a monetary policy rule (4.14) have to be satisfied by the equilibrium sequences $\{\widehat{c}_t, \widehat{\pi}_t, \widehat{m}_t, \widehat{R}_t\}_{t=0}^{\infty}$. In contrast to the **B**-version, consumption and inflation are independent of beginning-of-period real balances. To put it differently, predetermined real money balances can not serve as a state variable. Instead, the private sector behavior is entirely forward-looking in the **E**-version with the consequence that the equilibrium displays nominal indeterminacy under interest rate policy.

The following Proposition summarizes the conditions for equilibrium determination under interest rate policy.

Proposition 4.9. *Suppose that end-of-period money enters the utility function and that the central bank sets the nominal interest rate.*

1. *When current inflation enters the interest rate rule, $\widehat{R}_t = \rho_\pi\widehat{\pi}_t$, the equilibrium displays real determinacy and local stability if and only if $\rho_\pi > 1$.*
2. *When future inflation enters the interest rate rule, $\widehat{R}_t = \rho_\pi\widehat{\pi}_{t+1}$, inflation cannot uniquely be determined. The equilibrium sequences $\{\widehat{c}_t, \widehat{\pi}_{t+1}, \widehat{m}_t, \widehat{R}_t\}_{t=0}^{\infty}$ are locally stable and uniquely determined if and only if*

   (i) $\rho_\pi > 1$ *or* $\rho_\pi < \left(1 + \frac{2(z-1)\sigma_l\varepsilon_{ca}}{Y+\sigma_l(\varepsilon_{ca}+\sigma_a)}\right)^{-1}$ *for* $\sigma_l > 0$, *and*

   (ii) $\rho_\pi \neq 1$ *for* $\sigma_l = 0$.

Proof. See appendix 5.3.6.

As in the **B**-version, equilibrium determination depends on the particular interest rate rule. When the nominal interest rate is set contingent on current inflation, inflation can be determined for all periods. Under a forward looking interest rate policy, one can only uniquely determine future inflation. In any case, the initial price level and initial real balances are irrelevant for a *REE*, implying nominal indeterminacy and the absence of an endogenous state variable. Uniqueness of equilibrium sequences is further ensured by an active interest rate policy, $\rho_\pi > 1$, under both types of rules. For the special case, where the labor supply elasticity is infinite, any forward looking interest rate rule satisfying $\rho_\pi \neq 1$ leads to unique equilibrium sequences $\{\widehat{c}_t, \widehat{\pi}_{t+1}, \widehat{m}_t, \widehat{R}_t\}_{t=0}^{\infty}$.

Turning to the case where the central bank holds the money growth rate constant, I find that the equilibrium behavior closely relates to the one in the **B**-version.

Proposition 4.10. *Suppose that end-of-period money enters the utility function and that the money growth rate is held constant. Then, the equilibrium sequences* $\{\widehat{c}_t, \widehat{m}_t, \widehat{R}_t\}$ $\forall t \geq 0$ *and* $\{\widehat{\pi}_t\}$ $\forall t \geq 1$ *are locally stable and uniquely determined, and there exists a unique consistent price level* $\forall t \geq 0$*.*

Proof. See appendix 5.3.7.

Summing up, the specification of money demand has substantial consequences for the determination of equilibrium sequences and for macroeconomic stability. The beginning-of-period value for real money balances is only relevant for equilibrium determination in the **B**-version under a non-forward looking interest rate rule. In the **E**-version, where the households' behavior lacks any backward looking element, the initial value of real balances is irrelevant for any policy regime under consideration. Whether beginning-of period real money is serving as a relevant endogenous state variable or not, is, on the one hand, decisive for a unique determination of the price level, and, on the other hand, crucially affects the conditions for local stability and uniqueness under an interest rate policy regime: policy should rather be passive than active, to avoid unstable or oscillatory equilibrium sequences. Under a constant money growth regime, however, local stability and uniqueness of equilibrium sequences and a unique price level sequence is ensured for both versions – regardless of the labor supply elasticity. Given that the stock of money is known in every period, a unique sequence for real money balances suffices to pin down uniquely the entire sequence for the absolute price level. Evidently, this does not require the economy to evolve in a history dependent way.

4.3.3 Related results

The main novel results in this section refer to the case where beginning-of-period money enters the utility function and the central bank applies an interest rate rule, while some results for the alternative cases correspond to results in related studies on real balances effects and equilibrium determinacy in flexible price models.

For example, my findings for the **E**-version (see part 1 of Proposition 4.9) resemble the results in Benhabib et al. (2001a) and Woodford (2003) for non-separable utility functions. They find that when current inflation serves as an indicator, active interest rate setting is necessary and sufficient for local stability and uniqueness. This, however, changes when beginning-of-period money provides utility, since equilibrium sequences are then – except for the case $\sigma_l = 0$ – unstable or oscillatory (see Proposition 4.6). Thus, the literature has disregarded the role of predetermined real balances as a relevant state variable, which substantially affects the real and nominal determinacy properties.

If the monetary authority applies a constant money growth rule, then local stability and uniqueness impose restrictions on preferences only in case where the stock of money held at the beginning of the period provides utility. In particular, the inverse of the intertemporal elasticity of substitution for real money balances should not be too large (see Proposition 4.8), which corresponds to the results in Brock (1974), Matsuyama (1990),Carlstrom and Fuerst (2003) and Woodford (2003). Assuming that end-of-period money provides transaction services, Brock (1974), Matsuyama (1990) and Woodford (2003), show that local stability and uniqueness is ensured if consumption and real balances are Edgeworth-complements, as in my framework. Furthermore, Carlstrom and Fuerst (2003) find that the intertemporal elasticity of substitution for money can matter for local stability and uniqueness is guaranteed, as in Proposition 4.8.

To unveil the role of non-separability for the results and to facilitate comparisons with related studies (see, e.g., Carlstrom and Fuerst (2001), I further briefly discuss the case where money demand is separable, $\varepsilon_{ca} = \phi_{ac} = 0$. Then, the model reduces to

$$\widehat{R}_t = \widehat{\pi}_{t+1}, \text{ and } \sigma_a \widehat{m}_t = \begin{cases} -(z - \sigma_a)\,\widehat{R}_t & \text{for the } B\text{-version} \\ -(z-1)\,\widehat{R}_t & \text{for the } E\text{-version} \end{cases}.$$

while consumption is exogenously determined. When utility is separable, the conditions for uniqueness under money growth policy, which are presented in Proposition 4.8 and 4.10, are unchanged. In contrast to the results for the non-separable case, the particular stock of money that enters the utility function is now irrelevant for equilibrium determination under interest rate policy: Equilibrium uniqueness requires $\rho_\pi > 1$ for $\widehat{R}_t = \rho_\pi \widehat{\pi}_t$ and $\rho_\pi \neq 1$ for $\widehat{R}_t = \rho_\pi \widehat{\pi}_{t+1}$, which accords to the results in Carlstrom and Fuerst (2001). As in the case of non-separable utility, current inflation cannot be determined under a forward looking interest rate rule, while under a money growth rule inflation is only indetermined in the first period.

4.4 Imperfectly Flexible Prices

In a framework with monopolistic competitive firms and staggered price setting as developed by Calvo (1983), the initial price level belongs to the set of relevant state variables.[12] Under this specification, real balances serve as a relevant predetermined state variable for all aforementioned policy rules, when the beginning-of-period specification applies. If, however, the end-of-period stock of money enters the utility function, households are entirely forward looking, and real money serves as a relevant state variable only if monetary policy is history dependent, i.e., when the central bank applies a money growth rule. Nonetheless, the determinacy properties under constant money growth and sticky prices correspond to those under flexible prices. The main implications for equilibrium uniqueness and stability under imperfectly flexible prices are as follows:

- When beginning-of-period money provides utility, interest rate policy has to be passive to lead to locally stable, unique, and non-oscillatory equilibrium sequences, regardless whether current or future inflation enters the policy rule. An active interest rate policy is associated with locally stable and unique equilibrium sequences if and only if end-of-period money provides utility and current inflation serves as the policy indicator.[13]

- As under flexible prices, the central bank can ensure equilibrium sequence to be uniquely determined, locally stable, and non-oscillatory under both timing specifications by holding the growth rate of money constant, provided that real balance effects are not extremely large.

4.5 Conclusion

Real balance effects typically arise when transaction costs are specified in a general equilibrium model in form of shopping time or real resource costs, which are reduced by money holdings. The fact that the equilibrium sequences for real balances and consumption can then not separately be determined, is broadly viewed as negligible for the assessment of monetary policy, given that empirical evidence suggests

[12]Please refer to Schabert and Stoltenberg (2005) for details in that case.

[13]To be more precise, these results apply for finite labor supply elasticities. For the case of an infinite labor supply elasticity there is a related paper by Brückner and Schabert (2005). Assuming staggered price setting and a specific functional form for utility, they consider the implications of the timing of markets on optimal monetary policy under discretion.

real balance effects to be relatively small (Ireland 2004a). In contrast to this view, it is demonstrated in this paper that the existence (not the magnitude) of real balance effects has substantial implications for the determination of a rational expectations equilibrium and of the price level under interest rate policy.

However, for real balance effects to contribute to price level determination, as for example suggested by Patinkin (1949, 1965), predetermined real money balances have to serve as a state variable. Remarkably, these properties require that the stock of money at the beginning of the period yields transaction services, which corresponds to Svensson's timing assumption (1985) , that the goods market closes before the asset market opens. Hence, real money that has been acquired in the previous period restricts households' current consumption expenditures. Then, there exists a unique initial price level that is consistent with a rational expectations equilibrium, i.e., the equilibrium displays nominal determinacy. In that case, interest policy should be passive to ensure unique, non-oscillatory and locally stable equilibrium sequences – a violation of the Taylor-principle. If, on the other hand, current consumption is related to the end-of-period stock of money, then the equilibrium displays nominal indeterminacy, and the well-known principles for uniqueness and stability of equilibrium sequences of a cashless economy (roughly) apply. Remarkably, these results highlight, that the existence and timing of real balance effects jointly have substantial implications for equilibrium determination.

If monetary policy follows a constant money growth rule, the conditions for equilibrium uniqueness are likely to be ensured. Though the economy does not evolve in a history dependent way, the entire path for the absolute price level is uniquely determined in both versions. These results suggest that a central bank that aims to avoid multiple, unstable, or oscillatory equilibrium sequences in an environment where transaction frictions are non-negligible, should rather control the supply of money than the nominal interest rate.

Yet, an optimal conduct of monetary policy will certainly require the supply of money to be state contingent (as an interest rate feedback rule), which might be associated with different determinacy implications than a constant money growth regime.

5 Technical Appendix

5.1 Technical Appendix to chapter 2

5.1.1 Additional details for the proof of Theorem 2.1

First, we consider the perfect information environment. The Lagrangian of the optimal risk-sharing problem can be written as

$$\begin{aligned}\mathcal{L} = \frac{1}{4}\left(u(c^h_{1g}) + u(c^l_{1g}) + u(y^h) + u(y^l)\right) \\ + \frac{\beta}{8}\left(u(c^{hh}_{2g}) + u(c^{hl}_{2g}) + u(c^{lh}_{2g}) + u(c^{ll}_{2g}) + 4u(0)\right) \\ + \mu_1\left(\bar{y} - \frac{1}{2}\left(c^h_{1g} + c^l_{1g}\right)\right) + \mu_2\left(\bar{y} - \frac{1}{4}\left(c^{hh}_{2g} + c^{hl}_{2g} + c^{lh}_{2g} + c^{ll}_{2g}\right)\right) \\ + \lambda\left(u(c^h_{1g}) + \frac{\beta}{2}\left(u(c^{hh}_{2g}) + u(c^{hl}_{2g})\right) - u(y^h) - \frac{\beta}{2}\left(u(y^h) + u(y^l)\right)\right),\end{aligned}$$

where μ_1, μ_2, and λ denote the Lagrangian multipliers, $\bar{y} \equiv (y^h + y^l)/2$, and with consumption under type-b policy already substituted in.

The first order conditions with respect to c^h_{1g}, c^{hh}_{2g}, and c^{hl}_{2g} are

$$\frac{1}{4}u'(c^h_{1g}) - \mu_1\frac{1}{2} + \lambda u'(c^h_{1g}) = 0, \tag{5.1}$$

$$\frac{\beta}{8}u'(c^{hh}_{2g}) - \mu_2\frac{1}{4} + \lambda\frac{\beta}{2}u'(c^{hh}_{2g}) = 0, \tag{5.2}$$

$$\frac{\beta}{8}u'(c^{hl}_{2g}) - \mu_2\frac{1}{4} + \lambda\frac{\beta}{2}u'(c^{hl}_{2g}) = 0, \tag{5.3}$$

and with respect to c^l_{1g}, c^{lh}_{2g}, and c^{ll}_{2g} are

$$\frac{1}{4}u'(c^l_{1g}) - \mu_1\frac{1}{2} = 0, \tag{5.4}$$

$$\frac{\beta}{8}u'(c_{2g}^{lh}) - \mu_2\frac{1}{4} = 0, \tag{5.5}$$

$$\frac{\beta}{8}u'(c_{2g}^{ll}) - \mu_2\frac{1}{4} = 0. \tag{5.6}$$

The first order conditions can be analyzed in two steps. First, the optimal consumption in the second period is independent of the second period endowment realization. From (5.2) and (5.3) we immediately obtain $c_{2g}^{hh} = c_{2g}^{hl} \equiv c_{2g}^{h}$. Similarly, from (5.5) and (5.6) we get $c_{2g}^{lh} = c_{2g}^{ll} \equiv c_{2g}^{l}$. Thus, the second period resource constraint can be rewritten exactly as for the first period: $(c_{2g}^{h} + c_{2g}^{l})/2 = (y^h + y^l)/2$. Second, agents prefer to smooth consumption between the first and the second period. Substituting the Lagrange multipliers from (5.4) and (5.5) into (5.1) and (5.2) and combining with the resource constraints we get

$$\frac{u'(y^h + y^l - c_{1g}^{h})}{u'(c_{1g}^{h})} = \frac{u'(y^h + y^l - c_{2g}^{h})}{u'(c_{2g}^{h})},$$

From the two steps we conclude that the socially optimal consumption of the high endowment agents is constant over time

$$c_{1g}^{h} = c_{2g}^{hh} = c_{2g}^{hl},$$

except for the type-*b* policy when all goods are taxed away.

When the high endowment agent participation constraints (2.2) and (2.3) are binding, then

$$\begin{aligned} u(c_{1g}^{h}) &= \frac{1}{1+\beta}\left(u(y^h) + \frac{\beta}{2}\left(u(y^h) + u(y^l)\right)\right) \\ &= \frac{1+\beta/2}{1+\beta}u(y^h) + \frac{\beta/2}{1+\beta}u(y^l). \end{aligned} \tag{5.7}$$

Second, in the imperfect information environment the Lagrangian is

$$\begin{aligned} \mathcal{L} = \frac{1}{2}&\left(u(c_1^h) + u(c_1^l)\right) \\ &+ \frac{\beta}{8}\left(u(c_{2g}^{hh}) + u(c_{2g}^{hl}) + u(c_{2g}^{lh}) + u(c_{2g}^{ll}) + 4u(0)\right) \\ &+ \mu_1\left(\bar{y} - \frac{1}{2}\left(c_1^h + c_1^l\right)\right) + \mu_2\left(\bar{y} - \frac{1}{4}\left(c_{2g}^{hh} + c_{2g}^{hl} + c_{2g}^{lh} + c_{2g}^{ll}\right)\right) \\ &+ \lambda\left(u(c_1^h) + \frac{\beta}{4}\left(u(c_{2g}^{hh}) + u(c_{2g}^{hl})\right) - u(y^h) - \frac{\beta}{4}\left(u(y^h) + u(y^l)\right)\right). \end{aligned}$$

Similarly to perfect information, it follows from the first order conditions that

$$c_1^h = c_{2g}^{hh} = c_{2g}^{hl},$$

and when participation constraint (2.7) is binding

$$\begin{aligned} u(c_1^h) &= \frac{1}{1+\beta/2}\left(u(y^h) + \frac{\beta}{4}(u(y^h) + u(y^l))\right) \\ &= \frac{1+\beta/4}{1+\beta/2}u(y^h) + \frac{\beta/4}{1+\beta/2}u(y^l). \end{aligned} \tag{5.8}$$

Comparing (5.7) and (5.8) reveals that $u(c_{1g}^h) < u(c_1^h) < u(y^h)$, and taking into account $c_{1b}^h = y^h$ we obtain $c_{1g}^h < c_1^h < c_{1b}^h$.

5.1.2 Proof of Lemma 2.5

Let $S(\kappa)$ be the set of sustainable allocations. The outside option is always in the set of sustainable allocations, and the restrictions imposed by the participation constraints (2.27), (2.28) and consumption feasibility (2.29) define a bounded and closed set. Therefore, for any precision of the public signal, $S(\kappa)$ is nonempty and compact-valued. Furthermore, it can be shown that the correspondence $\varphi : [1/2; 1] \to \mathbb{R}_+^8$, which maps $\kappa \mapsto S(\kappa)$ is continuous. Given that the objective function (2.26) is also continuous, by the Theorem of the Maximum (Bergé 1963) there exists a solution to the optimal arrangement problem for any public signal precision, and the expected utility of the socially optimal arrangement is continuous in signal precision.

In addition, the set of sustainable allocations is convex-valued due to the concavity of the utility function, and the objective function is strictly concave. By the Maximum Theorem under Convexity the optimal arrangement is unique and continuous in signal precision.

5.1.3 Proof of Lemma 2.6

Let $\{c^i(\pi_j, s_k)\}$ be the optimal arrangement. First, we show that for any state (π_j, s_k) in the optimal arrangement the high income agents obtain at least as much as the low income agents, i.e. $c^h(\pi_j, s_k) \geq c^l(\pi_j, s_k)$. By contradiction, assume there exist inflation π_j and signal s_j such $c^h(\pi_j, s_k) < c^l(\pi_j, s_k)$. Let the perfect risk sharing allocation be defined as $\bar{x}(\pi_j) \equiv (x^h(\pi_j) + x^l(\pi_j))/2$, and consider an arrangement

$\{\tilde{c}^i(\pi_j, s_k)\}$ given by

$$\tilde{c}^h(\pi_j, s_k) = \tilde{c}^l(\pi_j, s_k) = \bar{x}(\pi_j), \quad \tilde{c}^i(\tilde{H}) = c^i(\tilde{H}),$$

with $\tilde{H}$ as the set of all possible states, excluding (π_j, s_k). By construction the arrangement provides strictly higher utility for risk-averse households than $\{c^i(\pi_j, s_k)\}$. We are left to prove that the arrangement $\{\tilde{c}^i(\pi_j, s_k)\}$ is sustainable. High income households undoubtedly accept the arrangement because it delivers both higher current period consumption and higher future arrangement utility. Low income households in state (π_j, s_k) obtain the same utility as the high income households. Given that the outside option is worse for low income households and the high income households accept the arrangement, the low income households must also accept the arrangement. Summing up, the arrangement $\{\tilde{c}^i(\pi_j, s_k)\}$ is sustainable and socially preferable over $\{c^i(\pi_j, s_k)\}$. This contradicts that $\{c^i(\pi_j, s_k)\}$ is the socially optimal arrangement.

Second, we show that for any state $c^h(\pi_j, s_k) \leq x^h(\pi_j, s_k)$. Again, by contradiction, assume that there is a state such that $c^h(\pi_j, s_k) > x^h(\pi_j, s_k)$. If the participation constraint for the high productivity agent under π_j inflation and s_k signal holds with equality then the future value of the arrangement is lower than the outside option value, and taking into account that for the low productivity agent from the resource constraint $c^l(\pi_j, s_k) < x^l(\pi_j, s_k)$ the participation constraint for the low productivity agent is violated. Therefore, the considered participation constraint for the high productivity agent can only hold with a strict inequality. Then, consider a consumption allocation $\{\tilde{\tilde{c}}^i(\pi_j, s_k)\}$ given by

$$\tilde{\tilde{c}}^h(\pi_j, s_k) = c^h(\pi_j, s_k) - \varepsilon, \quad \tilde{\tilde{c}}^l(\pi_j, s_k) = c^l(\pi_j, s_k) + \varepsilon, \quad \tilde{\tilde{c}}^i(\tilde{H}) = c^i(\tilde{H}).$$

By continuity there exists $\varepsilon > 0$ such that the consumption allocation $\{\tilde{\tilde{c}}^i(\pi_j, s_k)\}$ is sustainable, and by concavity the constructed allocation provides higher utility then the allocation $\{c^i(\pi_j, s_k)\}$, which contradicts that $\{c^i(\pi_j, s_k)\}$ is the socially optimal arrangement.

5.1.4 Proof of Proposition 2.9

The cutoff point for β is characterized by the tightest participation constraint, which is the one that becomes binding for any level of patience below the cutoff point.

Among the participation constraints only constraints for high productivity agents can be binding, which limits consideration to four cases.

There are two independent factors that determine the tightest participation constraint: the relative gain of deviation from perfect risk sharing to the outside option for high income households, and the expected future gain of perfect risk-sharing arrangement relative to the outside option. Without loss of generality, consider a case such that

$$u(x^h(\pi_l)) - u(\bar{x}(\pi_l)) \leq u(x^h(\pi_h)) - u(\bar{x}(\pi_h)) \tag{5.9}$$

$$u(\bar{x}(\pi_l)) - \frac{1}{2}(u(x^h(\pi_l)) + u(x^l(\pi_l))) < u(\bar{x}(\pi_h)) - \frac{1}{2}(u(x^h(\pi_h)) + u(x^l(\pi_h))), \tag{5.10}$$

where $\bar{x}(\pi_j) \equiv (x^h(\pi_j) + x^l(\pi_j))/2$ is the perfect risk-sharing allocation. The first inequality (5.9) states that the current period deviation for a high income household is more attractive in the high inflation state. The second inequality (5.10) implies that for the perfect risk-sharing arrangement the risk aversion effect dominates, i.e. the perfect risk-sharing arrangement provides higher value in comparison to the outside option under high inflation. Therefore, for any precision of the signal, the participation constraint of high productivity agents under high current inflation and a low future inflation signal is the one that imposes the tightest restriction. This constraint holds with equality at the cutoff point

$$\begin{aligned} u(\bar{x}(\pi_h)) - u(x^h(\pi_h)) + \bar{\beta}\kappa(\bar{V}_{rs}(\pi_l) - V_{at}(\pi_l)) \\ + \bar{\beta}(1-\kappa)(\bar{V}_{rs}(\pi_h) - V_{at}(\pi_h)) + \frac{\bar{\beta}^2}{1-\bar{\beta}}(\bar{V}_{rs} - V_{at}) = 0, \end{aligned} \tag{5.11}$$

where $\bar{V}_{rs}(\pi_j) = u(\bar{x}(\pi_j))$ and $\bar{V}_{rs} = (u(\bar{x}(\pi_h)) + u(\bar{x}(\pi_l)))/2$.

Solving (5.11), there exists a unique solution for $\bar{\beta}$ in $(0,1)$ due to $u(x^h(\pi_h)) - u(\bar{x}(\pi_h)) > 0$. Employing the implicit function theorem from (5.11) we get

$$\frac{d\bar{\beta}}{d\kappa} = \frac{\bar{\beta}(1-\bar{\beta})(\bar{V}_{rs}(\pi_h) - V_{at}(\pi_h) - \bar{V}_{rs}(\pi_l) + V_{at}(\pi_l))}{u(x^h(\pi_h)) - u(\bar{x}(\pi_h)) + dV(\kappa) + 2\bar{\beta}(dV(1/2) - dV(\kappa))} > 0,$$

where $dV(\kappa) \equiv \kappa(\bar{V}_{rs}(\pi_l) - V_{at}(\pi_l)) + (1-\kappa)(\bar{V}_{rs}(\pi_h) - V_{at}(\pi_h))$ and satisfies $0 \leq dV(\kappa) \leq dV(1/2)$.

5.1.5 Proof of Lemma 2.10

First, we show that if participation constraints are violated under perfect risk sharing for any state then the optimal consumption allocation satisfies

$$c^h(\pi_j, s_k) > \bar{x}(\pi_j) > c^l(\pi_j, s_k), \tag{5.12}$$

where $\bar{x}(\pi_j)$ again is the perfect risk sharing allocation in inflation state π_j. Without loss of generality, consider the participation constraint for households of high productivity in the previous period under currently high inflation that receive a high signal on future inflation. The constraint holds for the socially optimal arrangement

$$\begin{aligned} u(c^h(\pi_h, s_h)) + \beta(\kappa V_{rs}(\pi_h) + (1-\kappa)V_{rs}(\pi_l)) + \frac{\beta^2}{1-\beta}V_{rs} \geq \\ u(x^h(\pi_h)) + \beta(\kappa V_{at}(\pi_h) + (1-\kappa)V_{at}(\pi_l)) + \frac{\beta^2}{1-\beta}V_{at}, \end{aligned} \tag{5.13}$$

but is violated by assumption under perfect risk sharing

$$\begin{aligned} u(\bar{x}(\pi_h)) + \beta(\kappa u(\bar{x}(\pi_h)) + (1-\kappa)u(\bar{x}(\pi_l))) + \frac{\beta^2}{1-\beta}\bar{V}_{rs} < \\ u(x^h(\pi_h)) + \beta(\kappa V_{at}(\pi_h) + (1-\kappa)V_{at}(\pi_l)) + \frac{\beta^2}{1-\beta}V_{at}, \end{aligned} \tag{5.14}$$

where $\bar{V}_{rs} \equiv u(\bar{x}(\pi_h)) + u(\bar{x}(\pi_l))/2$ denotes the value of the perfect risk-sharing arrangement. The right hand side of (5.13) or (5.14) represents the total value of the outside option. Combining (5.13) and (5.14) we get

$$\begin{aligned} u(c^h(\pi_h, s_h)) + \beta(\kappa V_{rs}(\pi_h) + (1-\kappa)V_{rs}(\pi_l)) + \frac{\beta^2}{1-\beta}V_{rs} > \\ u(\bar{x}(\pi_h)) + \beta(\kappa u(\bar{x}(\pi_h)) + (1-\kappa)u(\bar{x}(\pi_l))) + \frac{\beta^2}{1-\beta}\bar{V}_{rs} \end{aligned} \tag{5.15}$$

Taking into account that the optimal contract delivers a value no larger than the value of perfect risk sharing $V_{rs}(\pi_j) \leq u(\bar{x}(\pi_j))$, and $V_{rs} \leq \bar{V}_{rs}$, from (5.15) we get

$$u(c^h(\pi_h, s_h)) > u(\bar{x}(\pi_h))$$

or, combining with resource feasibility

$$c^h(\pi_h, s_h) > \bar{x}(\pi_h) > c^l(\pi_h, s_h).$$

Similarly we can show that the same inequalities hold for the other public states.

Second, by contradiction, assume that there is one participation constraint for high productivity agents that is not binding. The Lagrangian of the optimal contract problem (2.26)-(2.29) can be written as

$$\mathcal{L} = (1 + \sum_{(\pi_j, s_k) \in \tilde{H}} \lambda^i(\pi_j, s_k))(u(c^h(\pi_j, s_k)) + u(c^l(\pi_j, s_k))) \\ + \mu(\pi_j, s_k)(c^h(\pi_j, s_k) + c^l(\pi_j, s_k) - 2\bar{x}(\pi_j)) + \xi(\tilde{H}), \quad (5.16)$$

where (π_j, s_k) is the state for which the participation constraint is not binding, $\tilde{H}$ is the set of all possible states, excluding (π_j, s_k), $\lambda^i(\pi_j, s_k)$ are the normalized Lagrange multipliers for the participation constraints, $\mu(\pi_j, s_k)$ are the Lagrange multipliers for resource constraints, and $\xi(\tilde{H})$ collects consumption and resources for states in $\tilde{H}$, and respective multipliers. The Lagrange multiplier for the participation constraint for state (π_j, s_k) is zero and is explicitly excluded from the Lagrangian.

Solving the optimal arrangement problem (5.16) we get

$$c^h(\pi_j, s_k) = c^l(\pi_j, s_k) = \bar{x}(\pi_j)$$

for the state (π_j, s_k), which contradicts the partial risk-sharing condition (5.12) stated above.

5.1.6 Proof of Lemma 2.11

The socially optimal risk sharing arrangement under uninformative signals can be analyzed as a fixed point problem in terms of the period value of the arrangement. When signals are uninformative, the optimal arrangement is conditional only on current inflation, and the number of binding participation constraints of high productivity households reduces to two.

The fixed point problem is constructed as follows. Let w be the future one-period value of an arrangement. We restrict attention to $w \in [V_{at}, \bar{w})$, since per assumption all participation constraints for high productivity agents are binding. The two binding participation constraints can be written as

$$u(c^h(\pi_j)) + \frac{\beta}{1-\beta} w = u(x^h(\pi_j)) + \frac{\beta}{1-\beta} V_{at} \quad \forall j, \quad (5.17)$$

and consumption feasibility is given by

$$c^h(\pi_j) + c^l(\pi_j) = x^h(\pi_h) + x^l(\pi_h) \quad \forall j. \tag{5.18}$$

The objective function of the optimal arrangement problem is

$$V(w) \equiv \frac{1}{4}\sum_{f,j} u(c^f(\pi_j)).$$

The optimal arrangement should necessary solve the fixed point problem $w = V(w)$.

We show that $V(w)$ is strictly increasing and strictly concave, therefore there exist at most two solutions to the fixed problem. From the participation constraints (5.17) and consumption feasibility constraints (5.18) it follows that $V(w)$ is strictly increasing

$$V'(w) = \frac{1}{4}\frac{\beta}{1-\beta}\left(-2 + \frac{u'(c^l(\pi_h))}{u'(c^h(\pi_h))} + \frac{u'(c^l(\pi_l)}{u'(c^h(\pi_l)}\right) > 0,$$

since perfect risk sharing is not sustainable per assumption. Strict concavity of $V(w)$ is implied by

$$\frac{d}{dw}\left(\frac{u'(c^l(\pi_j))}{u'(c^h(\pi_j))}\right) = \frac{\beta}{1-\beta}\frac{\left(u''(c^l(\pi_j)) + u''(c^h(\pi_j))\frac{u'(c^l(\pi_j))}{u'(c^h(\pi_j))}\right)}{\left(u'(c^h(\pi_j))\right)^2} < 0.$$

By construction, one solution to the fixed point problem is V_{at}. The concavity of $V(w)$ implies that the derivative of $V(w)$ at V_{at} is higher than at any partial risk-sharing allocation. Therefore, the derivative of $V'(w)$ at V_{at} must be greater than 1, which implies

$$\frac{1}{2}\left(\frac{u'(x^l(\pi_h))}{u'(x^h(\pi_h))} + \frac{u'(x^l(\pi_l))}{u'(x^h(\pi_l))}\right) > \frac{2-\beta}{\beta}$$

in order for the optimal arrangement to be different from the outside option.

From the other end, suppose there exists a socially optimal arrangement different from the allocation in the absence of transfers and participation constraints are binding. Then the value of this arrangement must be a solution to the fixed point problem. This requires that the slope of $V(w)$ at V_{at} must be necessarily larger than unity, due to the concavity of $V(w)$ and because the allocation in the absence of transfers must always be one solution of the constructed fixed point problem.

5.1.7 Proof of Theorem 2.12

Suppose $V_{rs}(\kappa_1) \leq V_{rs}(\kappa_2)$ for some $\kappa_1 < \kappa_2$. Consider an alternative consumption allocation $\{\tilde{c}^i(\pi_j, s_k, \kappa_1)\}$ for signal precision κ_1 constructed on the basis of the optimal allocation $\{c^i(\pi_j, s_k, \kappa_2)\}$ for κ_2 according to

$$\begin{aligned} u(\tilde{c}^h(\pi_j, s_h, \kappa)) & \\ &= -\beta\left(\kappa(V_{rs}(\pi_h, \kappa_2) - V_{at}(\pi_h)) + (1-\kappa)(V_{rs}(\pi_l, \kappa_2) - V_{at}(\pi_l))\right) \\ &\qquad + u(x^h(\pi_j)) - \frac{\beta^2}{1-\beta}(V_{rs}(\kappa_2) - V_{at}) \end{aligned} \tag{5.19}$$

$$\begin{aligned} u(\tilde{c}^h(\pi_j, s_l, \kappa)) & \\ &= -\beta\left((1-\kappa)(V_{rs}(\pi_h, \kappa_2) - V_{at}(\pi_h)) + \kappa(V_{rs}(\pi_l, \kappa_2) - V_{at}(\pi_l))\right) \\ &\qquad + u(x^h(\pi_j)) - \frac{\beta^2}{1-\beta}(V_{rs}(\kappa_2) - V_{at}) \end{aligned} \tag{5.20}$$

and the corresponding allocation for low productivity agents given by consumption feasibility. $V_{rs}(\pi_j, \kappa_2)$ and $V_{rs}(\kappa_2)$ characterize the optimal allocation for κ_2.

First, the alternative allocation $\{\tilde{c}^i(\pi_j, s_k, \kappa_1)\}$ is consumption-feasible by construction.

Second, the alternative allocation $\{\tilde{c}^i(\pi_j, s_k, \kappa_1)\}$ delivers strictly higher expected utility than the optimal allocation for signal precision κ_2, i.e. $\tilde{V}_{rs}(\kappa_1) > V_{rs}(\kappa_2)$, where $\tilde{V}_{rs}(\kappa) \equiv \frac{1}{8}\sum_{f,j,k} u(\tilde{c}^f(\pi_j, s_k, \kappa))$. We prove this result by showing that high productivity agents are indifferent between the optimal allocation and the alternative allocation, and low productivity agents strictly prefer the alternative allocation.

For signal precision κ_2 by assumption the risk aversion and wealth effects do not offset each other and the outside option is not the only sustainable arrangement. Without loss of generality, suppose that the risk aversion effect dominates, i.e. $V_{rs}(\pi_l, \kappa_2) - V_{at}(\pi_l) < V_{rs}(\pi_h, \kappa_2) - V_{at}(\pi_h)$. Subtracting (5.20) from (5.19) we get

$$\begin{aligned} u(\tilde{c}^h(\pi_j, s_h, \kappa)) - u(\tilde{c}^h(\pi_j, s_l, \kappa)) = (2\kappa - 1)\beta((V_{rs}(\pi_l, \kappa_2) - V_{at}(\pi_l)) \\ - (V_{rs}(\pi_h, \kappa_2) - V_{at}(\pi_h))). \end{aligned}$$

Therefore, for any $\kappa < \kappa_2$

$$u(c^h(\pi_j, s_h, \kappa_2)) < u(\tilde{c}^h(\pi_j, s_h, \kappa)) \leq u(\tilde{c}^h(\pi_j, s_l, \kappa)) < u(c^h(\pi_j, s_l, \kappa_2)). \qquad (5.21)$$

For high productivity agents, the alternative allocation for κ_1 and the optimal allocation for κ_2 deliver the same expected utility in any state π_j, as can be seen from adding (5.19) and (5.20).

For low productivity agents, the expected utility in state π_j, defined by

$$W^l(\pi_j, \kappa) \equiv \frac{1}{2} \sum_k u(\tilde{c}^l(\pi_j, s_k, \kappa))$$

is strictly decreasing in precision over $\kappa \leq \kappa_2$. This result follows from (5.19)-(5.21), consumption feasibility, and risk-aversion of agents:

$$\begin{aligned} \frac{\partial W^l(\pi_j, \kappa)}{\partial \kappa} &= -\frac{1}{2} \left(\frac{u'(\tilde{c}^l(\pi_j, s_h, \kappa))}{u'(\tilde{c}^h(\pi_j, s_h, \kappa))} - \frac{u'(\tilde{c}^l(\pi_j, s_l, \kappa))}{u'(\tilde{c}^h(\pi_j, s_l, \kappa))} \right) \times \\ &\quad (V_{rs}(\pi_l, \kappa_2) - V_{at}(\pi_l)) - (V_{rs}(\pi_h, \kappa_2) - V_{at}(\pi_h)) < 0 \quad \forall \kappa > 1/2, \end{aligned}$$

and $\partial W^l(\pi_j, 1/2)/\partial \kappa = 0$. In particular, this implies that $\tilde{V}_{rs}(\pi_j, \kappa_1) > V_{rs}(\pi_j, \kappa_2)$, and therefore $\tilde{V}_{rs}(\kappa_1) > V_{rs}(\kappa_2)$.

Third, the alternative allocation $\{\tilde{c}^i(\pi_j, s_k, \kappa_1)\}$ satisfies the participation constraints for signal precision κ_1. This results follows immediately from construction of the alternative allocation, and from the finding in the previous step that the alternative allocation for $\kappa_1 < \kappa_2$ provides strictly higher utility in all inflation states than the optimal allocation for κ_2.

Finally, the social value of the optimal allocation for κ_1 is at least as large as for any other feasible allocation compatible with participation constraints $V_{rs}(\kappa_1) \geq \tilde{V}_{rs}(\kappa_1)$. Therefore, $\tilde{V}_{rs}(\kappa_1) > V_{rs}(\kappa_2)$ is a contradiction to $V_{rs}(\kappa_1) \leq V_{rs}(\kappa_2)$.

5.2 Technical Appendix to chapter 3

5.2.1 Constraints and CRRA preferences in the steady state

Suppose that the utility function is of the CRRA form. Given an output subsidy that renders the steady state efficient, constraints (3.9)-(3.13), and the money demand equation can be combined to solve for Δ, y, c, l, R and m in terms of inflation.

$$\Delta = \frac{(1-\alpha)\rho^{\frac{\theta}{\theta-1}}}{1-\alpha\pi^{\theta}}, \tag{5.22}$$

with $\rho = (1-\alpha\pi^{\theta-1})/(1-\alpha)$,

$$y = [\frac{1}{a_2\Delta^{\omega}sc^{\sigma_c}}]^{\frac{1}{\sigma_c+\omega}}, \tag{5.23}$$

$$c = ysc, \tag{5.24}$$

$$l = y\Delta \tag{5.25}$$

$$R = \frac{\pi}{\beta}, \tag{5.26}$$

$$m = [R/(R-1)y^{\sigma_c}a_1 sc^{\sigma_c}]^{1/\sigma_m}. \tag{5.27}$$

5.2.2 Proof Proposition 3.3

Consider first the steady state utility if the inflation rate is zero. Correspondingly, gross inflation and price dispersion are 1, such that $y_{ZERO} = l_{ZERO}$. Using (5.23)-(5.27), one can compute $y_{ZERO} = 1/sc^{1/(1+\omega)} = l_{ZERO}$, $c_{ZERO} = sc^{\omega/(1+\omega)}$ and $m_{ZERO} = R_{ZERO}\eta_{R,ZERO}a_1 sc^{\omega/(1+\omega)}$. Then the period steady state utility of the average household is given by

$$u_{ZERO} = (1+a_1)\frac{\omega}{1+\omega}\ln(sc) - \frac{1}{(1+\omega)sc} - a_1\ln(1-\beta) + a_1\ln(a_1). \tag{5.28}$$

If $\pi = \beta + \epsilon$, then price dispersion is $\Delta_{FR} > 1$, and output equals
$y_{FR} = 1/(sc\Delta_{FR}^{\omega})^{1/(1+\omega)} < y_{ZERO}$, while $l_{FR} = (\Delta_{FR}/sc)^{1/(1+\omega)} > l_{ZERO}$.
Consumption and real money balances are then given by
$c_{FR} = (sc/\Delta_{FR})^{\omega/(1+\omega)} < c_{ZERO}$ and $m_{FR} = R_{FR}\eta_{R,FR}a_1(sc/\Delta_{FR})^{\omega/(1+\omega)} > m_{ZERO}$.

In this case, the period steady state utility is

$$u_{FR} = (1+a_1)\frac{\omega}{1+\omega}\ln(sc) - \frac{\omega}{1+\omega}(1+a_1)\ln(\Delta_{FR})...$$
$$- \frac{\Delta_{FR}}{(1+\omega)sc} + a_1 \ln(\frac{1+\beta^{-1}\epsilon}{\beta^{-1}\epsilon}) + a_1\ln(a_1). \quad (5.29)$$

Comparing (5.28) and (5.29), the Friedman rule yields higher utility as long as $a_1 > \underline{a_1}$. ■

5.2.3 Derivation of the aggregate supply curve in case of trend deflation

The first order condition for firms is given by (3.5), which reads in loglinearized terms:

$$\widehat{\frac{\widetilde{P}_{it}}{P_t}} = \widehat{F}_t - \widehat{K}_t. \quad (5.30)$$

Using the definition of the price level, $P_{it}/P_t = [(1-\alpha\pi_t^{\theta-1})/(1-\alpha)]^{1/(1-\theta)}$, the left hand side of (5.30) can be approximated to first order with

$$\frac{\alpha\bar{\pi}^{\theta-1}}{1-\alpha\bar{\pi}^{\theta-1}}\widehat{\pi}_t, \quad (5.31)$$

while the expression on the right-hand side can be approximated by (using the recursive form of F_t and K_t):

$$\widehat{F}_t = (1-\alpha\beta\bar{\pi}^{\theta})[u_c(\widehat{y_t - G_t}, \zeta_t) + \widehat{y}_t + \widehat{mc}_t] + \alpha\beta\bar{\pi}^{\theta}E_t(\theta\widehat{\pi}_{t+1} + \widehat{F}_{t+1}) \quad (5.32)$$

and

$$\widehat{K}_t = (1-\alpha\beta\bar{\pi}^{\theta-1})[u_c(\widehat{y_t - G_t}, \zeta_t) + \widehat{y}_t + (\widehat{1-\tau_t})]$$
$$+ \alpha\beta\bar{\pi}^{\theta-1}E_t((\theta-1)\widehat{\pi}_{t+1} + \widehat{K}_{t+1}). \quad (5.33)$$

Note that under trend deflation $\alpha\beta\bar{\pi} < 1$, while convergence has to be assumed, if one approximates around trend inflation (Ascari 2004). Therefore the difference

$\widehat{F}_t - \widehat{K}_t$ is given by:

$$\begin{aligned}\widehat{F}_t - \widehat{K}_t &= \alpha\beta\bar{\pi}^{\theta-1}(\bar{\pi}-1)\theta E_t\widehat{\pi}_{t+1} + \alpha\beta\bar{\pi}^{\theta-1}E_t\widehat{\pi}_{t+1} \\ &+ \alpha\beta\bar{\pi}^{\theta-1}E_t(\bar{\pi}\widehat{F}_{t+1} - \widehat{K}_{t+1}) + \alpha\beta\bar{\pi}^{\theta-1}(1-\bar{\pi})(u_c(\widehat{y_t - G_t}, \zeta_t) + \widehat{y}_t) \\ &+ (1-\alpha\beta\bar{\pi}^{\theta})\widehat{mc}_t - (1-\alpha\beta\bar{\pi}^{\theta-1})(\widehat{1-\tau_t}).\end{aligned}$$

Adding and subtracting $\alpha\beta\bar{\pi}^{\theta-1}E_t\widehat{F}_{t+1}$, then using (5.30) to substitute for $E_t(\widehat{F}_{t+1} - \widehat{K}_{t+1})$, $\widehat{F}_t - \widehat{K}_t$ and then for $\widehat{F}_{t+1}$ with (3.18), results in:

$$\begin{aligned}\widehat{\pi}_t &= \beta E_t\widehat{\pi}_{t+1} + \kappa^*\widehat{mc}_t + \widehat{F}_t\kappa^*\frac{(\bar{\pi}-1)}{1-\alpha\beta\bar{\pi}^{\theta}} \\ &+ \kappa^*(u_c(\widehat{y_t - G_t}, \zeta_t) + \widehat{y}_t)\frac{1-\bar{\pi}}{1-\alpha\beta\bar{\pi}^{\theta}} - \kappa^*\frac{1-\alpha\beta\bar{\pi}^{\theta-1}}{1-\alpha\beta\bar{\pi}^{\theta}}(\widehat{1-\tau_t}).\end{aligned}$$

Using that $u_c(\widehat{y_t - G_t}, \zeta_t) = -\sigma\widehat{y}_t + \sigma g_t$, and $\widehat{mc}_t = -(1+\omega)\widehat{a}_t + \sigma\widehat{y}_t + \omega\widehat{y}_t + \widehat{\mu}_t^w - \sigma g_t$, results in

$$\begin{aligned}\widehat{\pi}_t &= \beta E_t\widehat{\pi}_{t+1} + \widehat{F}_t\kappa^*\frac{(\bar{\pi}-1)}{1-\alpha\beta\bar{\pi}^{\theta}} + \kappa^*\widehat{y}_t[\sigma + \omega + \frac{(1-\bar{\pi})(1-\sigma)}{1-\alpha\beta\bar{\pi}^{\theta}}] \\ &+ \kappa^*\sigma g_t[\frac{1-\bar{\pi}}{1-\alpha\beta\bar{\pi}^{\theta}} - 1] + \kappa^*[\widehat{\mu}_t^w - (1+\omega)\widehat{a}_t + w_\tau\frac{1-\alpha\beta\bar{\pi}^{\theta-1}}{1-\alpha\beta\bar{\pi}^{\theta}}\widehat{\tau}_t].\end{aligned}$$

Applying the definition of $\widehat{y}_t^n$ and making use of $\widehat{\tau}_t = 0$ in this setting, results in (3.17) in the text.

5.2.4 Proof Proposition 3.4

The period utility function of the average household in equilibrium is given by:

$$\int_0^1 [u(y_t - G_t, \zeta_t) - v(l_{jt}) + z(m_t)]dj = u(y_t - G_t) + z(m_t) - \int_0^1 v(l_{jt})dj.$$

To derive (3.21) we need to impose that, in the optimal steady state, real money balances are sufficiently close to being satiated (see Woodford, 2003a, Assumption 6.1), the price dispersion associated with optimal inflation is sufficiently small, as well as that optimal inflation is close enough to one.

The first summands can be approximated to second order by:

$$u(y_t - G_t, \zeta_t) = u_c y[\widehat{y}_t + \frac{(1-\sigma)}{2}\widehat{y}_t^2 + \sigma g_t\widehat{y}_t] + t.i.s.p. + \mathcal{O}(\|\widehat{\xi}_t, \widehat{y}_t\|^3), \tag{5.34}$$

where we used that $(x_t - x) = x(\widehat{x}_t + 0.5\widehat{x}_t^2) + \mathcal{O}(\|\widehat{x}_t\|^3)$, t.i.s.p denotes terms independent of stabilization policy, $u_{c\zeta} = u_c$, $\zeta = 1$, $\sigma = \sigma_c sc^{-1}$, $\widehat{G}_t = (G_t - G)/y$, and that $g_t = \widehat{G}_t + \sigma^{-1}\widehat{\zeta}_t$. The utility of real money balances can be approximated by:

$$z(m) = u_c y[\frac{z_m m}{u_c y}\widehat{m}_t + 0.5\frac{z_m m}{u_c y}(1-\sigma_m)\widehat{m}_t^2] + t.i.s.p. + \mathcal{O}(\|\widehat{m}_t\|^3).$$

We treat $(R-1)/R$ as an expansion parameter, implying that $z_m/u_c - 0 = (R-1)/R - 0$ is at least of first order. Since we expand our model at a point near the zero bound, this means that the marginal utility of real money balances is close to zero.

Applying a first-order approximation to the money demand equation, and using that the coefficients $\sigma/\sigma_m = u_{cc}scy(R-1)/(Rz_{mm}m)$ and $s_m = z_m m/(u_c y)$ are of first order in $(R-1)/R$, we can approximate $z(m_t)$ as:

$$z(m_t) = -u_c y \frac{1}{2\sigma_m(R-1)v}\widehat{R}_t^2 + t.i.s.p. + \mathcal{O}(\|\widehat{\xi}_t, \widehat{y}_t, \widehat{R}_t\|^3), \tag{5.35}$$

where we assumed that $(R-1)/R - 0$ is of second order, implying that the linear term drops out in the quadratic approximation.

The third part of households' period utility can be approximated by:

$$v(l_t) = v_y y[\widehat{y}_t + \frac{1+\omega}{2}\widehat{y}_t^2 - (1+\omega)\widehat{a}_t\widehat{y}_t + \widehat{\Delta}_t] + t.i.s.p. + \mathcal{O}(\|\widehat{\xi}_t, \widehat{y}_t, \widehat{\Delta}_t^{0.5}, \varsigma\|^3), \tag{5.36}$$

with $v_y y = v_\Delta \Delta = v_l l$. This approximation is based on the assumption that

$$\int_0^1 (P_i/P)^{-\theta} di = (\check{P}/P)^{-\theta} - 1 = \mathcal{O}(\|\varsigma\|^3). \tag{5.37}$$

Here $\check{P}$ denotes the average long-term individual price and we collect in ς the distortions of the relative price due to price dispersion in the optimal steady state.[1] It follows that

$$\Delta_t - 1 = \frac{\theta}{2} var_i \ln(P_{it}) + \mathcal{O}(\|\widehat{p}_{it}, \widehat{\xi}_t, \varsigma\|^3),$$

and correspondingly $\widehat{\Delta}_t$ are of second order. Connecting (5.34),(5.35) and (5.36) by the relationship $v_y/u_c = amc/\mu^w = (1-\phi)$, with

$$\phi = 1 - \rho(\pi)^{\frac{1}{1-\theta}}(1-\tau)\frac{\theta-1}{\mu^w\theta}\frac{1-\alpha\beta\pi^\theta}{1-\alpha\beta\pi^{\theta-1}},$$

[1]This assumption depends on the absence of strategic complementarities in price setting (see Levin et al. (2006)). Then, price dispersion in the steady state is lower for deflation than for inflation (see Figure 4.)

results in:

$$U(c_t, l_t, m_t) = -u_c y[-\phi \widehat{y}_t + \frac{\sigma + \omega - \phi(1+\omega)}{2}\widehat{y}_t^2 - \widehat{y}_t(\sigma g_t + (1-\phi)(1+\omega)\widehat{a}_t)$$
$$+ (1-\phi)\frac{\theta}{2} var_i \ln(P_{it}) + \frac{1}{2\sigma_m (R-1)v}\widehat{R}_t^2] + t.i.s.p. + \mathcal{O}(\|\widehat{\xi}_t, \varsigma\|^3).$$

Using the sales tax as a sales subsidy by setting

$$1 - \tau = [\rho(\pi)^{\frac{1}{1-\theta}} \frac{\theta - 1}{\mu^w \theta} \frac{1 - \alpha\beta\pi^{\theta}}{1 - \alpha\beta\pi^{\theta-1}}]^{-1},$$

the linear term in the welfare approximation above vanishes:

$$U(t) = -\frac{u_c y}{2}[(\sigma+\omega)(\widehat{y}_t - \widehat{y}_t^*)^2 + \theta var_i \ln(P_{it}) + \frac{1}{\sigma_m(R-1)v}\widehat{R}_t^2]$$
$$+ t.i.s.p. + \mathcal{O}(\|\widehat{\xi}_t, \varsigma\|^3). \tag{5.38}$$

The variance of $\ln(P_t(i))$ is given by

$$var_i(\ln P_{it}) = \alpha var_i \ln(P_{it-1}) + \frac{\alpha}{1-\alpha}\widehat{\pi}_t^2 + t.i.s.p. + \mathcal{O}(\|\widehat{\xi}_t, \varsigma^{2/3}\|^3),$$

where we assumed that $\ln(\pi) = 0 + \mathcal{O}(\|\varsigma\|^2)$. Iterating the equation above forward starting from any $var_i \ln(P_{it_0-1})$ in the period before policy applies, for $t \geq t_0$ results in :

$$var_i \ln(P_{it}) = \sum_{s=t_0}^{t} \alpha^{t-s} \frac{\alpha}{1-\alpha}\widehat{\pi}_s^2 + t.i.s.p. + \mathcal{O}(\|\widehat{\xi}_t, \varsigma^{2/3}\|^3),$$

where we used that the initial price dispersion $var_i \ln(P_{it_0-1})$ is $t.i.s.p.$

Discounting, summing up and substituting for $var_i \ln(P_{it})$ in (5.38) results in (3.21) in Proposition 3.4. ■

5.2.5 Definition of the disturbances n_t, u_t and s_t

The exogenous fluctuations are defines as:

$$n_t = \eta_2 E_t(\widehat{a}_{t+1} - \widehat{a}_t) - \eta_1 \sigma E_t(g_{t+1} - g_t),$$

$$u_t = \eta_3 \widehat{a}_t + (1 - \alpha\beta\bar{\pi}^{\theta})\widehat{\mu}_t^w + \eta_3 \frac{\sigma}{1-\sigma} g_t$$

and

$$s_t = \eta_5 \widehat{a}_t + \eta_6 \sigma g_t + \kappa^* \widehat{\mu}_t^w.$$

The constants η_i, $i = 1, ..6$ are defined as:

$$\eta_1 = \frac{\omega}{\omega + \sigma},$$

$$\eta_2 = \frac{\sigma(1+\omega)}{\omega + \sigma},$$

$$\eta_3 = (1 - \alpha\beta\bar{\pi}^{\theta})(1+\omega)\frac{1-\sigma}{\omega + \sigma},$$

$$\eta_4 = \kappa^*[\omega + \sigma + \frac{(1-\bar{\pi})(1-\sigma)}{1 - \alpha\beta\bar{\pi}^{\theta}}],$$

$$\eta_5 = \frac{\eta_4(1+\omega)}{\omega + \sigma} - \kappa^*(1+\omega)$$

and

$$\eta_6 = \frac{\eta_4}{\omega + \sigma} + \kappa^*(\frac{1-\bar{\pi}}{1 - \alpha\beta\bar{\pi}^{\theta}} - 1).$$

5.2.6 The optimal policy problem from a timeless perspective

Under the timeless perspective the first order necessary conditions with respect to y_t, Δ_t, K_t, F_t, R_t and π_t for all $t \geq t_0$ are given by:

$$\begin{aligned} &u_c(t) - \Delta_t v_l(t)/a_t + z_m(t)m_c(t) + \lambda_{2t}(1-\tau)[u_{cc}(t)y_t + u_c(t)] \\ &+ \lambda_{3t}\mu_t^w[v_{ll}(t)\Delta_t y_t/a_t + v_l(t)] - \lambda_{5t}u_{cc}(t) + \lambda_{5t-1}\frac{u_{cc}(t)R_{t-1}}{\pi_t} \doteq 0 \end{aligned} \tag{5.39}$$

$$-y_t v_l(t)/a_t + \lambda_{3t}v_{ll}(t)y_t^2\mu_t^w + \lambda_{4t} - \lambda_{4t+1}\beta\alpha\pi_{t+1}^{\theta} \doteq 0 \tag{5.40}$$

$$-\lambda_{1t}\rho(t)^{\frac{1}{1-\theta}} - [\lambda_{2t} - \alpha\pi_t^{\theta-1}\lambda_{2t-1}] \doteq 0 \tag{5.41}$$

$$\frac{\theta}{\theta-1}\lambda_{1t} - [\lambda_{3t} - \alpha\pi_t^{\theta}\lambda_{3t-1}] \doteq 0 \tag{5.42}$$

$$z_m(t)m_R(t) + \lambda_{5t}\beta\frac{u_c(t+1)}{\pi_{t+1}} \leq 0, \tag{5.43}$$

and

$$\begin{aligned} &-\lambda_{1t}K_t\frac{\alpha}{1-\alpha}\pi_t^{\theta-2}\rho(t)^{\frac{\theta}{1-\theta}} + \lambda_{2t-1}\alpha K_t(\theta-1)\pi_t^{\theta-2} + \lambda_{3t-1}\alpha F_t\theta\pi_t^{\theta-1}... \\ &+ \lambda_{4t}[\theta\alpha\pi_t^{\theta-2}\rho(t)^{\frac{1}{\theta-1}} - \alpha\theta\pi_t^{\theta-1}\Delta_{t-1}] - \lambda_{5t-1}R_{t-1}\frac{u_c(t)}{\pi_t^2} \doteq 0. \end{aligned} \tag{5.44}$$

Note that λ_{2t_0-1}, λ_{3t_0} and λ_{5t_0-1} are the multiplier requiring initial commitment. The multipliers $\lambda_{1t} - \lambda_{5t}$ are associated with the following constraints:

$$\rho(\pi_t)^{\frac{1}{1-\theta}} K_t = \frac{\theta}{\theta - 1} F_t \tag{5.45}$$

$$K_t = u_c(y_t - G_t, \zeta_t)(1-\tau)y_t + \beta\alpha E_t K_{t+1}\pi_{t+1}^{\theta-1} \tag{5.46}$$

$$F_t = v_l(y_t\Delta_t/a_t)y_t\mu_t^w + \alpha\beta E_t F_{t+1}\pi_{t+1}^{\theta} \tag{5.47}$$

$$\Delta_t = (1-\alpha)\rho(\pi_t)^{\frac{\theta}{\theta-1}} + \alpha\Delta_{t-1}\pi_t^{\theta} \tag{5.48}$$

and

$$u_c(y_t - G_t, \zeta_t) = \beta R_t E_t \frac{u_c(y_{t+1} - G_{t+1}, \zeta_{t+1})}{\pi_{t+1}}, \tag{5.49}$$

with $\rho(\pi_t) \equiv (1 - \alpha\pi_t^{\theta-1})(1-\alpha)^{-1}$ for $R_t \geq 1$.

5.3 Technical Appendix to chapter 4

5.3.1 Equivalence between explicit transaction frictions and money-in-the-utility-function

In this appendix I examine the relation between the money-in-the-utility-function specification, which is applied throughout the analysis in chapter 4, and explicit specifications of transaction frictions, i.e., a shopping time specification and a specification where transactions are associated with real resource costs. For this demonstration, which relates to the analysis in Brock (1974) and Feenstra (1986), I assume for both alternative specification that the objective of the representative household is given by

$$\sum_{t=0}^{\infty} \beta^t v\left(c_t, x_t\right), \quad v_c > 0, v_{cc} < 0, v_x > 0,\ v_{cx} = 0, \text{ and } v_{xx} \leq 0, \tag{5.50}$$

where x denotes leisure.

1. First I consider a conventional *shopping time* specification which relates to the one applied in Brock (1974), McCallum and Goodfriend (1987) or Ljungqvist and Sargent (2004). For this purpose, I assume that households have to allocate total time endowment, which is normalized to equal one, to leisure x, working time l, and shopping time s, where the shopping time is assumed to depend on the consumption expenditures and on real balances

   $$1 \geq x_t + l_t + s_t, \quad \text{where} \quad s_t = H(c_t, A_t/P_t).$$

   Following Ljungqvist and Sargent (2004), I assume that the shopping time function H satisfies: $H_c > 0$, $H_{cc} > 0$, $H_a < 0$, and $H_{aa} > 0$ and $H_{ca} \leq 0$. Using that $x_t = 1 - l_t - s_t$ holds in the household's optimum, the utility function can be written as

   $$u(c_t, l_t, a_t) = v\left(c_t, 1 - l_t - H(c_t, a_t)\right),$$

   where $u_c = v_c + v_x(-H_c) \lesseqgtr 0$, $u_a = v_x(-H_a) > 0$, $u_l = -v_x < 0$, $u_{cc} = v_{cc} + v_{xx}H_c^2 - v_x H_{cc} < 0$, $u_{cl} = v_{xx}H_c \leq 0$, $u_{aa} = v_{xx}H_a^2 - v_x H_{aa} < 0$, $u_{al} = v_{xx}H_a \geq 0$, as well as $u_{ll} = v_{xx}$. Hence, the marginal utility of consumption, which is given by

$$u_{ca} = v_{xx} H_a H_c - v_x H_{ca},$$

is non-decreasing in real balances. If the shopping time function is non-separable or if leisure enters the utility function in a non-linear way, then marginal utility of consumption is strictly increasing in real balances.

2. Next, I closely follow the analysis in Feenstra (1986), and assume that purchases of consumption goods are associated with *real resource costs of transactions* $\phi(c_t, a_t)$, which satisfy: $\phi \geq 0$, $\phi(0, a) = 0$, $\phi_c > 0$, $\phi_a < 0$, $\phi_{cc} \geq 0$, $\phi_{aa} \geq 0$, $\phi_{ac} \leq 0$. Households' budget constraint then reads

$$M_t + B_t + P_t\phi(c_t, a_t) + P_t c_t \leq R_{t-1}B_{t-1} + M_{t-1} + P_t w_t l_t + P_t\omega_t - P_t\tau_t. \quad (5.51)$$

Maximizing (5.50) subject to (5.51), a no-Ponzi game condition, and $x_t \leq 1 - l_t$, leads – inter alia – to the following first order conditions for consumption and leisure:

$$\lambda_{rt}(1 + \phi_c(c_t, a_t)) = v_c(c_t), \qquad \lambda_{rt} w_t = v_x(1 - l_t),$$

where λ_{rt} denotes the Lagrange multiplier on (5.51). Note that the aggregate resource constraint now reads $y_t = c_t + \phi(c_t, a_t)$. Using the linear production technology, I therefore obtain the following equilibrium condition: $l_t = c_t + \phi(c_t, a_t)$. Combining these conditions and using that $w_t = 1$, leads to the following expression for the marginal utility of consumption:

$$v_c(c_t) = v_x(1 - c_t - \phi(c_t, a_t))(1 + \phi_c(c_t, a_t)).$$

Evidently, the equilibrium sequence of consumption is in general not independent from real money balances due to the existence of transaction costs. Differentiating the latter condition gives

$$\frac{dc_t}{da_t} = \frac{v_x\phi_{ca} - v_{xx}(1 + \phi_c)\phi_a}{v_{cc} + (1 + \phi_c)^2 v_{xx} - \phi_{cc} v_x}.$$

Hence, consumption is positively related to real balances even if either the cross-derivative ϕ_{ca} vanishes or the labor supply elasticity is infinite, i.e. $v_{xx} = 0$.

The corresponding properties of my MIU specification immediately show that an equivalence between the latter and the shopping time specification in 1. requires consumption and real balances to be Edgeworth-complements in the MIU version, if $v_{xx} < 0$ or $H_{ca} < 0$. In order to compare the MIU specification with the transaction cost specification in 2., I apply the first order condition for consumption and labor, the aggregate resource constraint, and the production function, which imply that the equilibrium sequence of consumption under a MIU specification satisfies $dc_t/da_t = -u_{ca}(u_{cc} + 1)u_{ll})^{-1}$. Evidently, an equivalence between both specifications requires consumption and real balances to be Edgeworth-complements, i.e. $u_{ca} > 0$, if $\phi_{ca} < 0$ or $v_{xx} < 0$. Thus, $v_{xx} < 0$, which implies a finite labor supply elasticity is sufficient for the existence of real balance effects under both specifications of transaction frictions.

5.3.2 Existence and uniqueness of the steady state

In this appendix, I briefly examine the steady state properties of the model. I restrict my attention to the case where the nominal interest rate is strictly positive, $R - 1 > 0$. I further omit, for convenience, bars which are throughout the paper used to mark steady state values.

When the stock of money at the beginning of the period enters the utility function, the deterministic steady state is characterized by the following conditions: $-u_l(c) = u_c(c, m/\pi)$, $R = \pi/\beta$ and $u_a(c, m/\pi)(u_c(c, m/\pi))^{-1} = R - 1$. For an interest rate policy regime, it is assumed that the policy rule of the central bank, $R(\pi)$, has a unique solution for the steady state relation $R = \pi/\beta$, so that the inflation rate can be substituted out. The first equation implies that c is an implicit function of m, $c = f(m)$, with $f'(m) = -u_{ca}[R\beta(u_{ll} + u_{cc})]^{-1} > 0$. Using this, the third equation can be used to determine the steady state value for m with $u_a(f(m), m/\pi)[u_c(f(m), m/(R\beta))]^{-1} = R - 1$. Differentiating the fraction on the left hand side reveals that

$$\frac{du_a/u_c}{dm} = \frac{u_c(u_{cc}u_{aa} - u_{ca}^2) + u_{ll}(u_{aa}u_c - u_a u_{ca})}{R\beta u_c^2(u_{ll} + u_{cc})} < 0,$$

as I assumed concavity for $u(c, a)$. It follows that a globally unique steady state exists if and only if:

$$\lim_{m \to 0} \frac{u_a(f(m), m/R\beta)}{u_c(f(m), m/(R\beta))} > R - 1.$$

Thus, steady state uniqueness relies on money to be essential (see Obstfeld and Rogoff, 1983): The marginal utility of real money balances should grow with a rate that is higher than the rate by which $1/u_c$ converges to zero when m approaches zero. An analogous line of arguments in case of a money growth policy leads to the condition $\lim_{m\to 0} u_a(g(m), m/\mu)[u_c(g(m), m/\mu)]^{-1} > \mu/\beta - 1$, where $c = g(m)$ is the implicit relation derived of the steady state condition $-u_l(c) = u_c(c, m/\mu)$ with $g'(m) = -u_{ca}[\mu(u_{ll} + u_{cc})]^{-1} > 0$. The condition for existence and uniqueness for the interest rate policy regime if end-of-period money provides transaction services is

$$\lim_{m\to 0} \frac{u_a(f_E(m), m)}{u_c(f_E(m), m)} > \frac{R-1}{R},$$

with $f_E(m)' = -u_{ca}[u_{ll} + u_{cc}]^{-1} > 0$. If the monetary authority applies a constant money growth rule then $\lim_{m\to 0} u_a(f_E(m), m)[u_c(f_E(m), m)]^{-1} > (\mu/\beta - 1)/(\mu/\beta)$ must be satisfied.

5.3.3 Proof of Proposition 4.6

Consider a monetary policy regime that sets the nominal interest rate contingent on changes in current inflation, $\widehat{R}_t = \rho_\pi \widehat{\pi}_t$.

First, I establish the conditions for local stability and uniqueness. Second, if the labor supply supply elasticity is finite, I show that the existence of exactly one stable eigenvalue (assigned to real money balances, η_m) implies non-zero coefficients η_i, $i = c, \pi, R$ of the fundamental solution.

Reducing the model in (4.11)-(4.13) leads to the following system in inflation and real money balances:

$$\begin{aligned}\begin{pmatrix} \widehat{\pi}_{t+1} \\ \widehat{m}_t \end{pmatrix} &= \begin{pmatrix} \frac{\sigma_l \varepsilon_{ca}}{\sigma_l+\sigma_c} + 1 & -\frac{\sigma_l \varepsilon_{ca}}{\sigma_l+\sigma_c} \\ \frac{\Upsilon+\sigma_l(\varepsilon_{ca}+\sigma_a)}{\sigma_c+\sigma_l} & -\frac{\Upsilon+\sigma_l(\varepsilon_{ca}+\sigma_a)}{\sigma_c+\sigma_l} \end{pmatrix}^{-1} \\ &\quad \times \begin{pmatrix} \frac{\sigma_l \varepsilon_{ca}}{\sigma_l+\sigma_c} + \rho_\pi & -\frac{\sigma_l \varepsilon_{ca}}{\sigma_l+\sigma_c} \\ z\rho_\pi & 0 \end{pmatrix} \begin{pmatrix} \widehat{\pi}_t \\ \widehat{m}_{t-1} \end{pmatrix} \\ &= \mathbf{A} \begin{pmatrix} \widehat{\pi}_t \\ \widehat{m}_{t-1} \end{pmatrix}.\end{aligned} \tag{5.52}$$

The characteristic polynomial of A can be simplified to

$$F(X) = X^2 - X\rho_\pi \frac{\Upsilon + \sigma_l(\varepsilon_{ca} + \sigma_a) - z\sigma_l\varepsilon_{ca}}{\Upsilon + \sigma_l(\varepsilon_{ca} + \sigma_a)} - \frac{\rho_\pi z \sigma_l \varepsilon_{ca}}{\Upsilon + \sigma_l(\varepsilon_{ca} + \sigma_a)}.$$

Consider the case the labor supply elasticity is finite $\sigma_l > 0$. In this case, the determinant of A, $\det(A) = F(0) < 0$, is strictly negative, indicating that exactly one eigenvalue is negative. Local stability and uniqueness then requires that there exists exactly one root of $F(X) = 0$ with modulus less than one. To examine the conditions for this, I use that $F(X)$ further satisfies

$$F(1) = 1 - \rho_\pi,$$
$$F(-1) = \frac{(1+\rho_\pi)(\mathrm{Y} + \sigma_l(\varepsilon_{ca} + \sigma_a)) - 2z\sigma_l\varepsilon_{ca}\rho_\pi}{\mathrm{Y} + \sigma_l(\varepsilon_{ca} + \sigma_a)}.$$

These conditions imply that for $F(1) > 0$ and $F(-1) < 0$), the model is locally stable, unique and non-oscillatory, since the stable eigenvalue is positive. If $F(1) < 0$ and $F(-1) > 0$, equilibrium sequences are locally stable and unique, but oscillatory, since the stable eigenvalue has a negative sign. Suppose that the real balance effect and that the inverse of the labor supply elasticity are large enough such that $\varepsilon_{ca} > \sigma_a(2z-1)^{-1}$ and $\sigma_l > \overline{\sigma_l}$, where $\overline{\sigma_l} \equiv \frac{\mathrm{Y}}{(2z-1)\varepsilon_{ca} - \sigma_a}$. Then, $F(-1)$ can be negative if ρ_π is sufficiently large. Local stability and uniqueness with $F(1) > 0$ and $F(-1) < 0$, is then ensured by moderate inflation elasticities satisfying $\overline{\rho_{\pi 1}} < \rho_\pi < 1$, where $\overline{\rho_{\pi 1}} \equiv \frac{\sigma_l(\varepsilon_{ca}+\sigma_a)+\mathrm{Y}}{\sigma_l(2z-1)\varepsilon_{ca} - \sigma_l\sigma_a - \mathrm{Y}}$. Alternatively, local stability and uniqueness arise for $F(1) < 0$ and $F(-1) > 0$, which requires $1 < \rho_\pi < \overline{\rho_{\pi 1}}$. Suppose that $\varepsilon_{ca} > \sigma_a(2z-1)^{-1}$ or $\sigma_l < \overline{\sigma_l}$. Then, $F(-1)$ cannot be negative and local stability and uniqueness then arise if $\rho_\pi > 1$.

So far I established the conditions for the existence of exactly one root η_m To establish the role of predetermined real money balances as a state variable, I need to show further that the coefficients η_i, $i = c, \pi, R$ are non-zero if η_m is non-zero and stable. Applying the method of undetermined coefficients to (4.11) results in the following restrictions for the coefficients η_c and η_π:

$$\eta_c = \frac{\varepsilon_{ca}(1-\eta_\pi)}{\sigma_c + \sigma_l},$$

implying that for $\varepsilon_{ca} > 0$ one not both coefficients can be zero. In particular, if η_π is neither zero nor 1, predetermined real money balances are a relevant endogenous state variable for $\rho_\pi \neq 0$. The money demand equation (4.13) implies that

$$\eta_\pi = -\frac{\eta_m k}{z\rho_\pi(\sigma_c + \sigma_l) - \eta_m k} \neq 0, 1$$

since $k = \mathrm{Y} + \sigma_l(\varepsilon_{ca} + \sigma_a) > 0$ due to strict concavity and $\rho_\pi \neq 0$ (no interest rate peg).

Now, consider the case where the labor supply elasticity is infinite, $\sigma_l = 0$. In this case $\det(A) = 0$, indicating that the beginning-of-period value for real money balances is irrelevant for the determination of $\widehat{\pi}_t$ and $\widehat{m}_t$. It follows that one eigenvalue equals zero and the other eigenvalue is larger than one, if and only if $\rho_\pi > 1$. Then, the equilibrium sequences for $\widehat{m}_t$, $\widehat{c}_{t+1}$, $\widehat{\pi}_t$ and $\widehat{R}_t$ for $t \geq 0$ are locally stable and uniquely determined, while $\widehat{c}_t$ cannot be determined. ■

5.3.4 Proof of Proposition 4.7

Consider a monetary regime in which future inflation serves as the policy indicator, $\widehat{R}_t = \rho_\pi E_t \widehat{\pi}_{t+1}$. Substituting for consumption with (4.11) and inserting the forward-looking feedback rule, the model in (4.11)-(4.13) can be reduced to

$$\begin{aligned}
\begin{pmatrix} \widehat{\pi}_{t+1} \\ \widehat{m}_t \end{pmatrix} &= \begin{pmatrix} \frac{\sigma_l \varepsilon_{ca}}{\sigma_l + \sigma_c} + 1 - \rho_\pi & -\frac{\sigma_l \varepsilon_{ca}}{\sigma_l + \sigma_c} \\ \frac{\Upsilon + \sigma_l(\varepsilon_{ca} + \sigma_a)}{\sigma_c + \sigma_l} - z\rho_\pi & -\frac{\Upsilon + \sigma_l(\varepsilon_{ca} + \sigma_a)}{\sigma_c + \sigma_l} \end{pmatrix}^{-1} \\
&\times \begin{pmatrix} \frac{\sigma_l \varepsilon_{ca}}{\sigma_l + \sigma_c} & -\frac{\sigma_l \varepsilon_{ca}}{\sigma_l + \sigma_c} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \widehat{\pi}_t \\ \widehat{m}_{t-1} \end{pmatrix} \\
&= \mathbf{B} \begin{pmatrix} \widehat{\pi}_t \\ \widehat{m}_{t-1} \end{pmatrix}.
\end{aligned} \tag{5.53}$$

The characteristic polynomial of $\mathbf{B}$ is given by

$$F(X) = X\left(X - \frac{\rho_\pi z \sigma_l \varepsilon_{ca}}{(\Upsilon + \sigma_l(\varepsilon_{ca} + \sigma_a))(1 - \rho_\pi) + \rho_\pi z \sigma_l \varepsilon_{ca}}\right).$$

Evidently, real money balances are not a relevant state variable, and one can only solve for $\widehat{m}_t$, $E_t\widehat{\pi}_{t+1}$, $E_t\widehat{c}_{t+1}$ and $\widehat{R}_t$ $\forall t \geq 0$. For a finite labor supply elasticity, $\sigma_l > 0$, local stability and uniqueness requires the other eigenvalue (one is equal to zero) to be unstable. A positive unstable root arises if monetary policy is active and $\sigma_l > \overline{\sigma_{l2}}$ or if $1 < \rho_\pi < \overline{\rho_{\pi 2}}$ for $\sigma_l < \overline{\sigma_{l2}}$ or $\varepsilon_{ca} < \sigma_a/(2z-1)$. A negative unstable root exists if $\rho_\pi > \overline{\rho_{\pi 2}}$, given that $\sigma_l > \overline{\sigma_l}$ and $\varepsilon_{ca} > \sigma_a/(2z-1)$, for $\sigma_l < \overline{\sigma_{l2}}$ or $\varepsilon_{ca} < \sigma_a/(2z-1)$. Thus, $1 < \overline{\rho_{\pi 2}} < \rho_\pi < -\overline{\rho_{\pi 1}}$ leads to a locally stable and unique equilibrium with a negative root for $\sigma_l < \overline{\sigma_l}$ or $\varepsilon_{ca} < \sigma_a/(2z-1)$. When the labor supply elasticity is infinite, $\sigma_l = 0$, then the Euler equation reads $(1-\rho_\pi)\widehat{\pi}_{t+1} = 0$. Thus, the model displays local stability and uniqueness if and only if $\rho_\pi \neq 1$. ■

5.3.5 Proof of Proposition 4.8

Under a constant money growth regime the nominal interest rate can be substituted out so that the reduced form system of the model in (4.11)-(4.13) reads (where I omitted the exogenous state)

$$\omega_1 \widehat{c}_{t+1} - (\omega_2 + 1)\, \widehat{m}_t + \omega_2 \widehat{\pi}_{t+1} = -\sigma_c \widehat{c}_t + \varepsilon_{ca} \widehat{m}_{t-1} - \varepsilon_{ca} \widehat{\pi}_t, \tag{5.54}$$

$$\varepsilon_{ca} \widehat{m}_{t-1} = (\sigma_l + \sigma_c)\, \widehat{c}_t + \varepsilon_{ca} \widehat{\pi}_t, \tag{5.55}$$

where $\omega_1 \equiv (\sigma_c(1-z) + \phi_{ac})\, z^{-1}$ and $\omega_2 \equiv (\varepsilon_{ca}(1-z) - z + \sigma_a)\, z^{-1}$, and $\widehat{m}_t = \widehat{m}_{t-1} - \widehat{\pi}_t$. After eliminating consumption with (5.55) and inflation with the linearized money growth rule (4.14), I get the following difference equation in $\widehat{m_t}$:

$$\widehat{m}_{t+1} = \frac{z(\sigma_l \varepsilon_{ca} + \sigma_l + \sigma_c)}{z(\sigma_l \varepsilon_{ca} + \sigma_l + \sigma_c) - (\Upsilon + \sigma_l \varepsilon_{ca} + \sigma_l \sigma_a)} \widehat{m}_t.$$

Once $\widehat{m_t}$ is determined, which requires an unstable root, one can solve for $\widehat{\pi}_t$ and $\widehat{c}_t$ $\forall t \geq 1$, while the initial values for consumption $\widehat{c}_0$ and inflation $\widehat{\pi}_0$ cannot be determined. Local uniqueness and stability of the equilibrium sequences $\{\widehat{m_t}, \widehat{\pi}_{t+1}, \widehat{c}_{t+1}, \widehat{R}_t\}_{t=0}^{\infty}$ thus require $\left| \frac{z(\sigma_l \varepsilon_{ca} + \sigma_l + \sigma_c)}{z(\sigma_l \varepsilon_{ca} + \sigma_l + \sigma_c) - (\Upsilon + \sigma_l \varepsilon_{ca} + \sigma_l \sigma_a)} \right| > 1$. If $z(\sigma_l \varepsilon_{ca} + \sigma_l + \sigma_c) - (\Upsilon + \sigma_l \varepsilon_{ca} + \sigma_l \sigma_a) > 0$, then the root is positive and unstable. Rearranging and using $\Upsilon = \sigma_c \sigma_a - \varepsilon_{ca} \phi_{ac}$ shows that this conditions is satisfied for $z > \sigma_a$. ■

5.3.6 Proof of Proposition 4.9

Consider the case where the central bank sets the nominal interest rate contingent on changes in current inflation, $\widehat{R}_t = \rho_\pi \widehat{\pi}_t$. After substituting for consumption and eliminating $\widehat{m}_t$ and $\widehat{m}_{t+1}$ with the static money demand equation (4.18), one obtains the following difference equation (where I omitted the exogenous state):

$$(d+1)\rho_\pi \widehat{\pi}_t = (d\rho_\pi + 1)\widehat{\pi}_{t+1},$$

where $d \equiv (z-1)\sigma_l \varepsilon_{ca} [\Upsilon + \sigma_l(\varepsilon_{ca} + \sigma_a)]^{-1} > 0$. Therefore $\rho_\pi > 1$ is necessary and sufficient for local stability and uniqueness of the equilibrium sequences of inflation $\widehat{\pi}_t$, real balances $\widehat{m_t}$, consumption $\widehat{c}_t$ and the nominal interest rate, $\widehat{R}_t \ \forall\, t \geq 0$.

Now, consider the case where future inflation serves as the policy indicator, $\widehat{R}_t = \rho_\pi \widehat{\pi}_{t+1}$. When the labor supply elasticity is finite, $\sigma_l > 0$, then the model in (4.16)-

(4.18) reduces to:

$$\widehat{\pi}_{t+2} = \frac{\rho_{\pi}(1+d)-1}{d\rho_{\pi}}\widehat{\pi}_{t+1}.$$

Evidently, one cannot determine current inflation rate $\widehat{\pi}_t$. One obtains a unique and locally stable solution for expected inflation, and the current values of consumption, real money balances and the nominal interest rate, if the eigenvalue of this equation is positive and unstable, which requires $\rho_{\pi} > 1$. Alternatively, $\rho_{\pi} < [1+2d]^{-1}$ ensures local stability and uniqueness, where one eigenvalue is smaller than -1. When the labor supply elasticity is infinite, $\sigma_l = 0$, then uniqueness of a equilibrium sequence for $\widehat{\pi}_{t+1}$ $\forall t \geq 0$ is guaranteed by $\rho_{\pi} \neq 1$.■

5.3.7 Proof of Proposition 4.10

Under a constant money growth policy, $\widehat{m}_t = \widehat{m}_{t-1} - \widehat{\pi}_t$, the model in (4.16)-(4.18) can – by eliminating the nominal interest rate – be reduced to:

$$\varepsilon_{ca}\widehat{m}_t = (\sigma_l + \sigma_c)\,\widehat{c}_t, \tag{5.56}$$

$$\gamma_1\widehat{c}_{t+1} + \gamma_2\widehat{\pi}_{t+1} + \gamma_3\widehat{m}_t = (\gamma_1 + \frac{\sigma_c + \phi_{ac}}{z})\widehat{c}_t, \tag{5.57}$$

where $\gamma_1 = \sigma_c(z-1)z^{-1} > 0$, $\gamma_2 = (1+\varepsilon_{ca})(z-1)z^{-1} > 0$ and $\gamma_3 = (\varepsilon_{ca} + \sigma_a)z^{-1} > 0$. Eliminating consumption with (5.56) and inflation with the linearized money growth rule leads to the following difference equation in real money balances:

$$\widehat{m}_{t+1} = \frac{[\sigma_l(1+\varepsilon_{ca})+\sigma_c](z-1) + \Upsilon + \sigma_l(\varepsilon_{ca}+\sigma_a)}{[\sigma_l(1+\varepsilon_{ca})+\sigma_c](z-1)}\widehat{m}_t,$$

which evidently exhibits an unstable root. Thus, one can uniquely determine end-of-period real balances $\widehat{m_t}$, current consumption $\widehat{c}_t$, the nominal interest rate $\widehat{R}_t$ $\forall t \geq 0$, while inflation $\widehat{\pi}_t$ can only be determined for $t \geq 1$. ■

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Selbständigkeitserklärung

Hiermit erkläre ich, die vorliegende Arbeit selbständig ohne fremde Hilfe verfasst und nur die angegebene Literatur und Hilfsmittel verwendet zu haben. Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Amsterdam, den 20.September 2009 Christian Stoltenberg